115. *Boundary Behaviour of Functions Harmonic in the Unit Ball*

By Shinji YAMASHITA
Mathematical Institute, Tôhoku University

(Comm. by Kinjirô KUNUGI, M. J. A., June 12, 1970)

1. The main purpose of this note is to prove Meier's theorem ([5], Satz 5, cf. [2], p. 154) in a real-harmonic form in the open unit ball U whose centre is the origin O in the Euclidean space R^3.

We begin with definitions of cluster sets following the planar cases (cf. [2], [6]). The two-point compactification $R^1 \cup \{-\infty, +\infty\}$ of the real number system R^1 is denoted by R^*. Let Ω be a domain in R^3, Q be a point of the boundary $\partial\Omega$ and $\mathcal{G}$ be a subset of Ω whose closure $\overline{\mathcal{G}}$ in R^3 contains Q. Let $f(P)$ be a real-valued function in Ω. Then, the cluster set of f at Q along $\mathcal{G}$ is defined by

$$C_{\mathcal{G}}(f, Q) = \bigcap_{r>0} \overline{f(\delta_r \cap \mathcal{G})},$$

where δ_r is the open ball $\{P; \overline{PQ} < r\}$ and the closure is taken in R^*. By a *cone* $\Delta = \Delta(Q, \varphi, h)$ (in Ω) at Q we mean an open circular cone in Ω with vertex Q, axis along a straight line through Q, generating angle (=one half of the opening angle) $\varphi, 0 < \varphi < \pi/2$, and altitude h. A *segment* X (in Ω) at Q is an open rectilinear segment X in Ω terminating at Q. The cluster sets corresponding to $\mathcal{G} = \Omega, \Delta$ and X will be denoted by $C_\Omega(f, Q), C_\Delta(f, Q)$ and $C_X(f, Q)$ respectively; these sets are non-empty and closed in R^* and in the case where f is continuous, they are, except possibly for $C_\Omega(f, Q)$, connected, i.e., of a form of "interval" $[a, b]$, $a, b \in R^*$.

A point $Q \in \partial\Omega$ is called a *Plessner point* of f if for any cone Δ at Q, $C_\Delta(f, Q) = R^*$. A *Fatou point* $Q \in \partial\Omega$ of f is a point at which $\bigcup_{\Delta} C_\Delta(f, Q)$ consists of a single point of R^*; here, Δ ranges over all cones at Q. A point $Q \in \partial\Omega$ is called a *Meier point* of f if $\bigcap_X C_X(f, Q) = C_\Omega(f, Q) \neq R^*$, where X ranges over all segments at Q. The totality of Plessner (Fatou, Meier, resp.) points of f will be denoted by $I(f, \Omega)$ $(F(f, \Omega), M(f, \Omega)$, resp.).

Our main theorem is stated in the case where Ω is the ball.

Theorem 1. *Let f be harmonic in the ball $U = \{P; \overline{OP} < 1\}$. Then*

$$\partial U \setminus \{I(f, U) \cup M(f, U)\}$$

is of first category in Baire's sense on the unit sphere ∂U.

Meier's theorem is usually called "topological analogue of