114. On D-dimensions of Algebraic Varieties

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The purpose of this note is to give an outline of our recent results on D-dimensions of algebraic varieties. Details will be published elsewhere.[1)]

Letting k denote an algebraically closed field of characteristic zero, we shall work in the category of schemes over k. Let V be a complete algebraic variety of dimension n, and let D be a divisor on V. We denote by $l(D)-1$ the dimension of the complete linear system $|D|$ associated with D. We consider the set of all positive integers m satisfying $l(mD)>0$, which we indicate by $\mathbf{N}(D)$. Assume that $\mathbf{N}(D)$ is not empty. Then $\mathbf{N}(D)$ forms a sub-semigroup of the additive group of all integers. Hence, letting $m_0(D)$ be the g.c.d. of the integers belonging to $\mathbf{N}(D)$, we can find a positive integer $N(D)$ such that $m\in \mathbf{N}(D)$ if $m\equiv 0$ mod. $m_0(D)$, $m\geqq N(D)$.

Theorem 1. *There exist positive numbers α, β and a non-negative integer κ such that the following inequality holds for every sufficiently large integer m:*

$$\alpha m^{\kappa}\leqq l(mm_0(D)D)\leqq \beta m^{\kappa}.$$

It is easy to check that κ is independent of the choice of α and β. *We define the D-dimension of V to be the integer κ*, provided that $l(mD)>0$ for at least one positive integer m. We denote the D-dimension of V by $\kappa(D,V)$. In the case in which $l(mD)=0$ for every positive integer m, *we define the D-dimension of V to be* $-\infty$: $\kappa(D,V)=-\infty$.

Theorem 2. *Assume that $\kappa(D,V)>0$. For an arbitrarily fixed integer p which is greater than a constant depending on D, there exists a positive number γ such that the following inequality holds for every sufficiently large integer m:*

$$l(mm_0(D)D)-l(\{mm_0(D)-pm_0(D)\}D)\leqq \gamma m^{\kappa-1}, \quad \kappa=\kappa(D,V).$$

We recall that, in classical algebraic geometry, the index of an algebraic system on an algebraic variety of dimension n is defined to be the number of those distinct members of the system which pass through r independent generic points of V, where $r=$ the dimension of the system + the dimension of its member $-n+1$.

Theorem 3. *Suppose that $\kappa=\kappa(D,V)$ is positive. Then there*

1) On D-dimensions of algebraic varieties (to appear in Journal of Mathematical Society of Japan).