111. *Rings in which Every Maximal Ideal is generated by a Central Idempotent*[*)]

By Kiyoichi OSHIRO
Department of Mathematics, Yamaguchi University

(Comm. by Kenjiro SHODA, M. J. A., June 12, 1970)

Introduction. Recently, in his paper [2], M. Satyanarayana has proved that, for a commutative ring R with identity 1, the following conditions are equivalent:

(1) *R is a finite direct sum of fields.*

(2) *Every maximal ideal is generated by an idempotent.*

(3) *Every maximal ideal is a direct summand of R.*

(4) *Every maximal ideal is R-projective as a right R-module and is principally generated by a zero-divisor.*

(5) *Every proper maximal ideal is R-injective as a right R-module.*

(6) *R has no nilpotents and every proper maximal ideal has a non-zero annihilator.*

By using the technique of the sheaf theory as in [1], we shall extend the above result to a non-commutative case.

In this paper all rings R are assumed to possess an identity element 1, and all R-modules are unitary modules. The term "ideals" will always mean "two-sided ideals".

1. Preliminaries. Pierce [1] defined, for each ring R, a sheaf $S(R)$ of rings over a Boolean space $X(R)$ (that is, a totally disconnected compact Hausdorff space) in such a way that R is the ring of global cross sections of $S(R)$.

Let $B(R)$ be the Boolean ring consisting of all central idempotents of R and let $X(R)$ be the *Spec* $B(R)$ consisting of all prime ideals of $B(R)$. Let x be a point in $X(R)$. Then, for each element e in x, there is a neighborhood of x, namely $U_e(x)=\{y \in X(R) \mid e \in y\}$. These neighborhoods form a base of the open sets of $X(R)$ and with this topology $X(R)$ becomes a Boolean space. Note that the neighborhood $U_e(x)$ is an open-closed set of $X(R)$.

For x in $X(R)$, we denote R/Rx, by R_x, where Rx is the ideal of R generated by x. Define $S(R)=\bigcup_{x\in X(R)} R_x$. Let $\pi: S(R)\to X(R)$ be given by the condition $\pi^{-1}(x)=R_x$. For $r\in R$ and $x\in X(R)$, let $\sigma_r(x)$ be the image of r under the natural homomorphism of R onto R_x.

*) Dedicated to Professor K. Asano for the celebration of his sixtieth birthday.