152. *On the Convergence Criteria of Fourier Series*

By Masako IZUMI, Shin-ichi IZUMI and V. V. Gopal RAO

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1. Introduction and Theorems.

1.1. We consider functions f which are even, integrable on the interval $(0,\pi)$ and are periodic with period 2π. The Young-Pollard convergence criterion of Fourier series of f reads as follows [1]:

Theorem YP. *If*

$$\int_0^t f(u)du=o(t) \quad as \quad t\to 0 \tag{1}$$

and

$$\int_0^t |d(uf(u))| \leqq At \quad as \quad t\to 0, \tag{2}$$

then the Fourier series of f converges at the origin.

This was generalized by H. Lebesgue [1] (or [2]) in the following form:

Theorem L. *If the condition* (1) *holds and*

$$\int_t^\pi |f(u)-f(u+t)|u^{-1}du=o(1) \quad as \quad t\to 0, \tag{3}$$

then the Fourier series of f is convergent at the origin.

Later this was further generalized by S. Pollard [1]:

Theorem P. *If the condition* (1) *holds and*

$$\lim_{k\to\infty}\limsup_{t\to 0}\int_{kt}^\pi |f(u)-f(u+t)|u^{-1}du=0, \tag{4}$$

then the Fourier series of f is convergent at the origin.

Does there exist any convergence criterion which contained in Theorem P but not in Theorem L? If there exists, Theorem P is properly more general than Theorem L. One of the object of this paper is to give an answer to this question.

1.2. On the other hand, M. and S. Izumi [3] proved the following

Theorem I. *If* (1) *holds and*

$$\int_t^\pi |d(u^{-a}f(u))| \leqq At^{-a} \quad as \quad t\to 0 \tag{5}$$

for an a, $0<a<1$, then the Fourier series of f converges at the origin.

Later B. Kwee [4] proved the

Theorem K. *The condition* (2) *implies* (5) *and the condition* (5) *implies* (4). *That is, Theorem I contains Theorem YP as a particular case, but is contained in Theorem P as a particular case.*

The question arises: what is the relation between Theorems L and