193. On Closed Graph Theorem

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This paper is to give a type of closed graph theorem for topological linear spaces similar to the one discussed in the previous paper [4], generalizing and simplifying the results obtained in [1], [2], and [3].

We make use of the notations in [4].

A filter Φ in a linear space E is said to be a *LS-filter* if Φ is generated by the complements of all the finite union of linear subspaces E_n $(n=1,2,\cdots)$ such that $E=\bigcup_{n=1}^{\infty} E_n$.

A subset A of a linear space E is said to be *linearly* open if for any straight line L in $E, L\cap A$ is *open* in L by its usual topology.

A filter Φ in a linear space E is said to be a *P-filter* if for every x in E there exists a linearly open set A such that either A is disjoint from Φ or Φ_A, considered as a filter in E, is finer than a LS-filter. (In general, we identify a filter Ψ in a subset of E with a filter in E generated by Ψ.)

A linear topological space E (in the sequel we suppose that every linear topological space is Hausdorff) is called a *generalized netted space* (called GN-space in the sequel) if there exists a sequence of P-filters Φ_n $(n=1,2,\cdots)$ such that every ultrafilter Ψ with $\Psi \not\supset \Phi_n$ $(n=1, 2,\cdots)$ converges in E.

E is called a *pre-GN-space* if there exists a sequence of P-filters Φ_n $(n=1,2,\cdots)$ such that every ultrafilter Ψ with $\Psi \not\supset \Phi_n$ $(n=1,2,\cdots)$ is a Cauchy-filter in E. The P-filters Φ_n, in these cases, are called defining filters for E.

Let φ be a linear mapping from a linear space E into a linear space F. The image $\varphi(A)$ of a linearly open subset A of E by φ is linearly open in $\varphi(E)$ and the inverse image $\varphi^{-1}(B)$ of a linearly open subset B in $\varphi(E)$ by φ is linearly open in E.

If φ is an one-to-one linear mapping from E into F, then the image $\varphi(\Phi)$ of a P-filter Φ in E is a P-filter.

If φ is a linear mapping from E into F, then the inverse image $\varphi^{-1}(\Phi)$ of a P-filter Φ in F such that $\varphi(E)$ is not disjoint from Φ is a P-filter in E. In particular, if E is a linear subspace of F, then for every P-filter Φ in F such that E is not disjoint from Φ, Φ_E is a P-filter in E, and a P-filter in E can be considered as a P-filter in F.

By virtue of these facts, we can see easily that the class of GN-