192. On Some Theorems of Berberian and Sheth

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1. Introduction. In this paper an operator T means a bounded linear operator acting on a complex Hilbert space H.

Following Halmos [4] we define the numerical range $W(T)$ as follows:

$$W(T)=\{(Tx,x)\,;\,\|x\|=1\}.$$

The basic facts concerning $W(T)$ are that it is convex and that its closure $\overline{W(T)}$ contains the spectrum $\sigma(T)$ of T.

Definition 1 ([4]). An operator T is said to be *convexoid* if

$$\overline{W(T)}=\mathrm{co}\,\sigma(T)$$

where the bar denotes the closure and co $\sigma(T)$ means the convex hull of the spectrum $\sigma(T)$ of T.

It is known that hyponormal operator is convexoid.

S. K. Berberian introduced the notion "*cramped*" of the unitary operator as follows:

Definition 2 ([1]). An unitary operator is said to be *cramped* if its spectrum is contained in some semicircle of the unit circle

$$\{e^{i\theta}\,;\,\theta_1\leqq\theta\leqq\theta_2,\,\theta_2-\theta_1<\pi\}.$$

Definition 3. A closed sector $\mathcal{S}$ is said to be *cramped sector if*

$$\mathcal{S}=\{re^{i\theta}\,;\,r\geqq 0,\,\theta_1\leqq\theta\leqq\theta_2,\,\theta_2-\theta_1<\pi\}$$

and $\theta_2-\theta_1$ is named to be *sector angle* of cramped sector $\mathcal{S}$.

Two lines are said to be the sector lines respectively which start the origin through the end point of the semicircle of the cramped sector, that is to say, a cramped sector consists of two sector lines and a semicircle. [cf., L_1, L_2 of Fig.]

Definition 4 ([2] [7]). An operator T is said to satisfy the condition G_1 if

$$\|(T-\lambda)^{-1}\|\leqq[\mathrm{dist}\,(\lambda,\sigma(T))]^{-1}$$

for all $\lambda\notin\sigma(T)$.

Definition 5 ([9]). A point α of $\sigma(T)$ is a *semibare point* if it lies on the circumference of some closed disk which contains no other point of $\sigma(T)$.

The set of all semibare points of $\sigma(T)$ will be denoted by $SB(\sigma(T))$. We decompose $T=UR$, polar decomposition of T. In this paper we

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