214. On Multi-Valued Mappings and Generalized Metric Spaces

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Quite a few different kinds of spaces like 1-st countable spaces, sequential spaces and M-spaces have been characterized as images or inverse images of metric spaces by unique-valued mappings satisfying certain conditions, but so far little efforts have been made to take advantage of multi-valued mappings in this aspect of study. The purpose of the present paper is to show usefulness of such mappings in characterization of two interesting classes of generalized metric spaces, σ-spaces (due to [4]) and M^*-spaces (due to [2]). Throughout the paper spaces are at least T_1. As for general terminologies and symbols the reader is refered to [3]. Some results, terminologies and references concerning multivalued mappings will be found in [1]. We also use Theorem 1 of [5] in the following discussions.

Definition 1. Let f be a multi-valued mapping from a space X to a space Y such that $f(x)\neq\emptyset$ for every $x\in X$, and $f^{-1}(y)\neq\emptyset$ for every $y\in Y$. (Such a mapping will be called simply a *map* from on.) Then for each subset C of X and for each subset D of Y we define the following symbols.

$$f(C)=\cup\{f(x)\mid x\in C\},\ f\langle C\rangle=\{y\mid y\in Y, f^{-1}(y)\subset C\},$$
$$f^{-1}(D)=\cup\{f^{-1}(y)\mid y\in D\},\ f^{-1}\langle D\rangle=\{x\mid x\in X, f(x)\subset D\}.$$

Then f is called *closed* if $f(C)$ is closed in Y for every closed in Y for every closed subset C of X. If for each $y\in Y$ there is $x\in f^{-1}(y)$ such that $f^{-1}\langle V\rangle$ is a nbd(=neighborhood) of x for every nbd V of y, then the map f is called *selection continuous.* If for each $y\in Y$ and for every nbd V of y there is $x\in f^{-1}(y)$ such that $f^{-1}\langle V\rangle$ is a nbd of x, then f is called *w. selection continuous.* It is obvious that for one-valued mappings our definition of closed map turns out to be the ordinary one, and 'selection continuous' as well as 'w. selection continuous' coincide with 'continuous' in the ordinary sense. Furthermore f will be called a *s. perfect map* if it is closed, selection continuous and compact, i.e. $f^{-1}(y)$ is compact for every $y\in Y$.

Proposition 1. *Let X be a regular σ-space and Y a space. If there is a closed, w. selection continuous map f from X to Y, then Y is σ.*