212. *Notes on Regular Semigroups. II*

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First we give a new characterization of regular semigroups.[1)]

Theorem 1. *A semigroup S is regular if and only if the relation*

(1) $$L \cap R = RSL$$

holds for every left ideal L and every right ideal R of S.

Proof. Let S be a regular semigroup. Then the well known characterization due to L. Kovács and K. Iséki implies that

(2) $$L = SL$$

for any left ideal L of S, and similarly we have

(3) $$R = RS$$

for any right ideal R of S. (2) and (3) imply

(4) $$L \cap R = SL \cap RS = (RS)(SL) = RSL,$$

i.e., the condition (1) is necessary.

Conversely, let S be a semigroup with property (1) for any left ideal L and any right ideal R of S. To show that S is regular, let a be an arbitrary element of S. Then (1) implies

(5) $$a \in L(a) \cap R(a) = R(a)SL(a) \subseteq aSa,$$

that is, S is a regular semigroup.

Next we give a similar characterization of semigroups which are semilattices of groups.[2)]

Theorem 2. *A semigroup S is a semilattice of groups if and only if the relation*

(6) $$L \cap R = LSR$$

holds for every left ideal L and every right ideal R of S.

Proof. Let S be a semigroup which is a semilattice of groups. It is known that every one-sided ideal of S is two-sided and S is regular (see [1] or [4]). This implies that

(7) $$SI = I = IS$$

holds for any ideal I of S. Hence we get

(8) $$I_1 \cap I_2 = I_1 S \cap SI_2 = I_1 S I_2$$

for any couple of (two-sided) ideals of S, i.e. the condition (6) holds.

Conversely, let S be a semigroup with property (6) for any left ideal L and any right ideal R of S. Then (6) implies that $L = LS^2$ and

1) For the notation and terminology we refer to [1].

2) For other characterizations of semigroups which are semilattices of groups, see [3]–[5].