19. *The Implicational Fragment of R-mingle*

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The relevant logic R was first defined in Belnap [1] though the implicational fragment of R which we refer to as RI in this note goes back to Church's weak implication [2]. Kripke [3] constructed "Sequenzen-kalkül" equivalent to RI. Anderson and Belnap [4] and the author [5] gave systems of the natural deduction equivalent to RI. By adding a mingle axiom $\alpha\supset(\alpha\supset\alpha)$ to R, we get a system R-mingle RM (defined by Meyer and Dunn [6]). Here the mingle axiom has the effect of Gentzen type "mingle" rule introduced by Ohnishi and Matsumoto [7].

In this note we shall give a system of the natural deduction equivalent to RMI, that is, the implicational fragment of RM. And then we shall show that the cut elimination theorem holds in Sequenzen-kalkül equivalent to RMI. Finally we shall give the decision procedure for RMI.

(A) The calculus RMI.

(Aa) Axioms.

Let α, β, γ be arbitrary formulae.

(Aa1) $((\alpha\supset\alpha)\supset\beta)\supset\beta$.

(Aa2) $(\alpha\supset\beta)\supset((\beta\supset\gamma)\supset(\alpha\supset\gamma))$.

(Aa3) $(\alpha\supset(\alpha\supset\beta))\supset(\alpha\supset\beta)$.

(Aa4) $\alpha\supset((\alpha\supset\alpha)\supset\alpha)$.

(Aa5) $\alpha\supset(\alpha\supset\alpha)$.

(Ab) Provability.

(Ab1)–(Ab5) Each of the axioms, (Aa1)–(Aa5), is provable in RMI.

(Ab6) If α and $\alpha\supset\beta$ are provable in RMI, then β is provable in RMI. This rule is called modus ponens (MP).

We shall abbreviate the statement "α is provable (in RMI)" to "$(RMI)\vdash\alpha$".

(Ac) Derived rules and theorems.

Let $A_n(\xi)$ denote the formula $\alpha_n\supset(\cdots\supset(\alpha_1\supset\xi)\cdots)$, where $A_0(\xi)$ means the formula ξ. Let $B_m(\xi)$ denote $\beta_m\supset(\cdots\supset(\beta_1\supset\xi)\cdots)$, where $B_0(\xi)$ means ξ.

(Ac1) $\vdash\alpha\supset\alpha$.

(Ac2) If $\vdash\alpha\supset\beta$ and $\vdash\beta\supset\gamma$, then $\vdash\alpha\supset\gamma$.

(Ac3) If $\vdash\alpha\supset\beta$ and $\vdash\gamma\supset((\alpha\supset\beta)\supset\delta)$, then $\vdash\gamma\supset\delta$.