15. *A Uniqueness Theorem for Symmetric Hyperbolic Systems of First Order in One Space Variables*

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1. In [4], Segal has shown that a solution of the relativistic wave equation $u_{tt}-u_{xx}+\alpha u=0$, $\alpha>0$, which vanishes on the forward light rays $x^2-t^2=0$, $t>0$, vanishes identically under certain boundedness condition. This is quite different from the ordinary wave equation where one has a wide class of outgoing waves which vanish in a light cone. This result has been extended by Goodman [1] and Morawetz [3] to the equation $u_{tt}-\Delta u+\alpha u=0$ in three space variables. The condition on the solution is that energy integral $\int_{R^3}(|\nabla u|^2+|u_t|^2+\alpha|u|^2)dv$ is bounded. Recently, Taniguchi [5] has given another proof of the above result in one space variable under the stronger assumption of initial values and also proved the similar result for some first order symmetric hyperbolic systems in one space variable. This paper is intended to extend the result for hyperbolic systems in [5].

Let us consider the uniqueness of the Cauchy problems for hyperbolic systems of first order:

$$\begin{cases} \dfrac{\partial u}{\partial t}=A\dfrac{\partial u}{\partial x}+iBu \\ u(0,x)=u_0(x) \end{cases} \tag{1.1}$$

in a half space $\{(t,x)\,|\,t\geqq 0, x\in R^1\}$ where A and B are $N\times N$-constant symmetric matrices, and u is an N vector ($N\geqq 2$).

We assume the following condition throughout this paper:

(I) $\det(\lambda I-(A\xi+B\eta))$ has real distinct zeros for any real $(\xi,\eta)\neq(0,0)$.

Then we obtain

Theorem. *Let u be a solution of* (1.1) *for $u_0(x)\in\mathcal{D}^1_{L^2}$. If $u(t,x)=0$ on $x+a_1t=0$ and $x+a_Nt=0$, where a_1 and a_N are the maximum and the minimum eigenvalues of matrix A, then $u(t,x)\equiv 0$.*

In [5], we assumed the condition: $\det(\lambda I-(A\xi+B\eta))$ has a form $\prod_{i=1}^{s}[\lambda^2-d_i^2(\xi^2+\eta^2)]$ for any real $(\xi,\eta)\neq(0,0)$, where d_i are positive and distinct, and $N=2S$. In this sense, this paper is an extension of the result in [5].

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