6. *Construction of a Local Elementary Solution for Linear Partial Differential Operators.* I

By Takahiro KAWAI
Research Institute for Mathematical Sciences, Kyoto University

(Comm. by Kunihiko KODAIRA, M. J. A., Jan. 12, 1971)

Let $P(x, D_x)$ be a partial differential operator with real analytic coefficients. Assume that the principal part P_m of P is simple characteristic and that P_m is of real coefficients. The purpose of this note is to construct $E(x, y)$ which satisfies $P(x, D_x)E(x, y)=\delta(x-y)$ near (x_0, x_0, ξ_0) as sections of the sheaf $\mathcal{C}$, where ξ_0 is a cotangent vector at x_0. (We refer the reader to Sato [7], [8] about the notion of the sheaf $\mathcal{C}$ defined on the cotangential sphere (or co-sphere) bundle. See also Kashiwara and Kawai [3]). In other words we construct a local co-spherical elementary solution for $P(x, D_x)$. We construct $E(x, y)$ in two different methods. The first one relies on the analysis in a complex domain and the second on the theory of pseudo-differential operators of finite type developed in Kashiwara and Kawai [3]. The extension of our theory to the operators with complex coefficients will be given in our forthcoming note. The details of this note will be published elsewhere. (See also Kawai [5].)

1°. We begin with the following Theorem 1 essentially due to Hamada [1], which treats the singular Cauchy problem in a complex domain.

Let $P(z, D_z)$ be a linear partial differential operator with holomorphic coefficients defined near the origin of C^n and have the form $P(z, D_z)=\sum_{j=0}^{m} a_j(z, D_{z'})\partial^j/\partial z_1^j$, where $a_0(z, D_z)\equiv 1$, $z'=(z_2, \cdots, z_n)$ and $a_j(z, D_{z'})$ is a differential operator of order at most $(m-j)$.

Denote by $P_m(z, \xi)$ the principal symbol of $P(z, D_z)$ where ξ is a cotangent vector at z which stands for D_z. Assume further that one of the solutions $\xi_1^0(z\,;\xi_2, \cdots, \xi_n)$ of $P_m(z\,;\xi_1, \xi_2, \cdots, \xi_n)=0$ is holomorphic in (z, ξ') and that $\partial/\partial\xi_1(P_m(z\,;\xi_1^0, \xi_2, \cdots, \xi_n))\neq 0$ near $(z, \xi')=(0, \xi_0')$, where $\xi'=(\xi_2, \cdots, \xi_n)$.

We denote by $\varphi(z, \xi'\,; s, y')$ the phase function with parameter $(s, y')=(s, y_2, \cdots, y_n)$ corresponding to ξ_1^0, that is, the characteristic function of $P(z, D_z)$ satisfying

(i) $P_m(z, \operatorname{grad}_z\varphi)\equiv 0$

(ii) $\varphi(s, z', \xi', s, y')=\langle z'-y', \xi'\rangle$

(iii) $\operatorname{grad}_z\varphi|_{z_1=s}=(\xi_1^0(s, z'\,;\xi_2, \cdots, \xi_n), \xi_2, \cdots, \xi_n)$.

Then we have