5. *A Remark on the Meet Decomposition of Ideals in Noncommutative Rings**)

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Introduction. In his paper [4] N. Radu has called that a commutative ring R is in the class $\mathfrak{D}$ if every ideal of R is represented as an intersection of primary ideals of R, and has shown that if R is in the class $\mathfrak{D}$, then $CB+A=C+A$ holds for ideals A, B and C of R such that $C\subseteq \bigcap_{\alpha\in I_B}(A+B_\alpha)$, where $\{B_\alpha \mid \alpha\in I_B\}$ is the set of all ideals which have the same nilradical with that of B.

The aim of this note is to generalize the above fact to noncommutative rings. Throughout this note, R is a noncommutative ring. The existence of unity is not assumed. The term *ideals* mean *two-sided ideals*, and (x) means the principal ideal generated by an element x. An ideal Q of R is called a (*right*) *M-primary* [n-primary] ideal if $AB\subseteq Q$ and $A\not\subseteq Q$, for ideals A and B, imply that B is contained in the McCoy's [nilpotent] radical of Q. The *right residual* of an ideal A by an ideal B is denoted by $A:B$, that is, $A:B=\{x\in R \mid xB\subseteq A\}$. A ring R will be called that it is in *the class* $\mathfrak{D}$ *with respect to the McCoy's* [*nilpotent*] *radical* if every ideal of R is represented as an intersection of M-primary [n-primary] ideals of R.

§1. Throughout this note, $\bar{A}$ will denote *the McCoy's radical* of an ideal A of R, that is, $\bar{A}$ is the intersection of all minimal prime ideals containing A. For an ideal B, I_B will mean the set of the indices of the ideals B_α with $\bar{B}_\alpha=\bar{B}$.

Lemma 1. *The following conditions are equivalent*:

(1) *R is in the class $\mathfrak{D}$ with respect to the McCoy's* [*nilpotent*] *radical.*

(2) *Every strongly meet irreducible ideal is M-primary* [*n-primary*].

Proof. This is immediate from the fact that every ideal is represented as an intersection of strongly meet irreducible ideals.

Theorem 1. *The following conditions are equivalent*:

(1) *R is in the class $\mathfrak{D}$ with respect to the McCoy's radical.*

(2) *If A, B and C are ideals such that $C\subseteq \bigcap_{\alpha\in I_B}(A+B_\alpha)$ then $CB+A=C+A$.*

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*) Dedicated to Professor K. Asano on his sixtieth birthday.