4. *A Note on Locally Uniform Rings and Modules*

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In [3] and [4], A. W. Goldie has investigated the structure of closed right ideals and annihilator right ideals of (semi)-prime right Goldie rings and has obtained interesting results. We generalize, in Section 1, Goldie's results on closed right ideals and annihilator right ideals of (semi)-prime right Goldie rings to those of right stable rings in the sense of [8]. In second section we shall give "density theorem" in basic uniform modules. Concerning the terminology we refer to [9].

1. On closed right ideals of right stable rings. Let M be a faithful right R-module. A submodule U is said to be uniform iff $U\neq 0$ and every pair of nonzero submodules of U has a nonzero intersection. A submodule K is said to be closed if it has no essential extensions in M. Clearly K is closed iff K is a complemented submodule in the sense of Goldie [4]. An R-module M is said to be locally uniform if every nonzero submodule of M contains a uniform submodule.

Proposition 1. *Let M be a faithful locally uniform right R-module and let K be a closed submodule of M. Then K is an intersection of maximal closed submodules of M* (cf. [4], Theorem 1.5).

Proof. Let K be a relative complement of a submodule L (cf. [4]). Then, there exists an independent set $\{A_i\}$ of uniform submodules such that $L' \supset \Sigma_i \oplus A_i$. We set $N_i = K \oplus \Sigma_{j\neq i} \oplus A_j$ for each i, then $N_i \cap A_i = 0$. Choose a maximal closed submodule N_i^* such that $N_i^* \supseteq N_i$ and $N_i^* \cap A_i = 0$ for each i. If $(\bigcap_i N_i^*) \cap (\Sigma_i \oplus A_i) \neq 0$, then there exist $\{A_i\}_{i=1}^n$ such that $(N_1^* \cap \cdots \cap N_n^*) \cap (A_1 \oplus \cdots \oplus A_n) \neq 0$. On the other hand we have $(N_1^* \cap \cdots \cap N_n^*) \cap (A_1 \oplus \cdots \oplus A_n) = 0$, which is shown by repeated application of the modular law. Hence $(\bigcap_i N_i^*) \cap (\Sigma_i \oplus A_i) = 0$ and $K = \bigcap_i N_i^*$, as desired.

Following R. E. Johnson [8], R is said to be a right stable ring iff R is a right locally uniform ring with $Z_r(R)=0$ and $(\Sigma A)^r = 0$, where A runs over all uniform right ideals. An element u of R is said to be uniform iff uR^1 is a uniform right ideal, where uR^1 is the principal right ideal generated by u (cf. [4]).

Proposition 2. *If R is a right stable ring, then a right ideal M is a maximal right annihilator ideal if and only if $M = u^r$ for some uniform element u of R. In particular, u^r is maximally closed.*

Proof. The "only if" part is immediate by Theorem 6.9 of [7].