1. *Even Maps from Spheres to Spheres*

By Juno MUKAI
College of General Education, Osaka University

(Comm. by Kenjiro SHODA, M. J. A., Jan. 12, 1971)

1. Introduction. The n-sphere S^n is the set of vectors in Euclidean space R^{n+1} having unit length. An even map f from S^n to a topological space X is a continuous map preserving base points which satisfies $f(-x)=f(x)$ for any $x \in S^n$.

In this note we deal with the general problem of representing homotopy classes by even maps from spheres to spheres.

To state the results, we denote by $\widetilde{KO}^*$ the functor in the real K-theory [2]. Suppose $k\equiv 0, 1, 2$ or $4 \bmod 8$, then we have

Theorem 1.1. *An element α of the homotopy group $\pi_{n+k}(X)$ of a finite CW-complex X which induces non-zero homomorphism α^*: $\widetilde{KO}^n(X)\to\widetilde{KO}^n(S^{n+k})$ $(\approx Z$ or $Z_2)$ can not be represented by any even map in the following cases:*

i) $n\equiv 2 \bmod 4$ *if* $k\equiv 1 \bmod 8$,

ii) $n\equiv 0$ or $3 \bmod 4$ *if* $k\equiv 2 \bmod 8$,

iii) $n\equiv 0 \bmod 2$ *if* $k\equiv 0 \bmod 4$.

By the methods of H. Toda and J. F. Adams, we have a family of the elements $\mu_{s,n}$ of $\pi_{n+k}(S^n)$ if $k=8s+1$ and $n\geq 3$. We note that $\mu_{0,n}$ is the $(n-2)$-fold suspension $\eta_n=S^{n-2}\eta_2$, where η_2 is the homotopy class of the Hopf map from S^3 to S^2.

Corollary 1.2. *Suppose $k=8s+1$ and $n\geq 3$, then*

i) *$\mu_{s,n}$ can not be represented by any even map if $n\not\equiv 2 \bmod 4$,*

ii) *$\mu_{s,n}\eta_{n+k}$ can not be represented by any even map if $n\equiv 0$ or* $3 \bmod 4$.

By Theorem 2 of [8], η_{n-1} can not be represented by any polynomial map from S^n to S^{n-1} if n is a power of 2. Since a form of even degree is an even map, Corollary 1.2 partially generalizes the above result of R. Wood.

We denote by ι_n the homotopy class of the identity of S^n and by ν_n the generator of the 2-component of $\pi_{n+3}(S^n)\approx Z_{24}$ for $n\geq 5$.

Theorem 1.3. i) *Suppose $n+k\equiv 2 \bmod 4$, then $\alpha\eta_{n+k}$ and $\alpha\eta^2_{n+k}$ are represented by even maps for any $\alpha\in\pi_{n+k}(S^n)$ respectively.*

ii) *Suppose $n+k\equiv 1 \bmod 4$ and $n\geq k+5$ and let $\alpha\in\pi_{n+k}(S^n)$ be of order 2, then we have the following.*

a) *Any element of the Toda bracket $\{\alpha, 2\iota_{n+k}, \eta_{n+k}\}$ is represented by an even map.*