# 41. On Axiom Systems of Ontology. II

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(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1971)

It is well known that Leśniewski's original system of Ontology has the form of the following single axiom [1], [2].

T. $a \varepsilon b \equiv [\exists c]\ \{c \varepsilon a\} \wedge [c]\{c \varepsilon a \supset c \varepsilon b\} \wedge [cd]\{c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d\}.$

It is mentioned that the following four theses are inferentially equivalent to {A1, A2, A3, A4} by C. Lejewski [1].

A 1. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\}$

A 2. $(a \varepsilon b \wedge c \varepsilon a) \supset c \varepsilon b$

A 3. $a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$

A 4. $c \varepsilon a \wedge [d]\{d \varepsilon a \supset d \varepsilon b\} \wedge [de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset a \varepsilon b$

In this paper, we shall prove that T and {A1, A2, A3, A4} are equivalent. Furthermore, we shall prove that A1 and A2 alone can serve as axiom system of Ontology.

**Lemma 1.** T *implies* A1, A2, A3 *and* A4.

The proof will be given in the form of suppositional proofs [1], [2].

T 1=A 1. $a \varepsilon b \supset [\exists c]\{c \varepsilon a\}$ (T)

T 2=A 2. $a \varepsilon b \wedge c \varepsilon a \supset c \varepsilon b$

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|---|---|---|---|
| Proof. | 1 | $a \varepsilon b$ | (premise) |
| | 2 | $c \varepsilon a \supset$ | |
| | 3 | $[c]\{c \varepsilon a \supset c \varepsilon b\}$ | (T, 1) |
| | 4 | $c \varepsilon a \supset c \varepsilon b$ | (OΠ: 3) |
| | | $c \varepsilon b$ | (4, 2) |

T 3=A 4. $c \varepsilon a \wedge [d]\{d \varepsilon a \supset d \varepsilon b\} \wedge [de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset a \varepsilon b$

| | | | |
|---|---|---|---|
| Proof. | 1 | $c \varepsilon a$ | (premise) |
| | 2 | $[d]\{d \varepsilon a \supset d \varepsilon b\}$ | |
| | 3 | $[de]\{d \varepsilon a \wedge e \varepsilon a \supset d \varepsilon e\} \supset$ | |
| | 4 | $[\exists c]\{c \varepsilon a\}$ | (DΣ: 1) |
| | | $a \varepsilon b$ | (T, 4, 2, 3) |

D 1. $x \varepsilon a^*b \equiv x \varepsilon a \wedge b \varepsilon x$ (rule of adding definition)

T 4=A 3. $a \varepsilon b \wedge c \varepsilon a \wedge d \varepsilon a \supset c \varepsilon d$

| | | | |
|---|---|---|---|
| Proof. | 1 | $a \varepsilon b$ | (premise) |
| | 2 | $c \varepsilon a$ | |
| | 3 | $d \varepsilon a \supset$ | |
| | 4 | $a \varepsilon b^*c$ | (1, 2, D1) |
| | 5 | $d \varepsilon b^*c$ | (3, 4, T2) |
| | | $c \varepsilon d$ | (D1, 5) |