64. On the α-Deficiency of Meromorphic Functions under Change of Origin

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1. Introduction. Let $f(z)$ be a transcendental meromorphic function in $|z|<\infty$ of order $\rho, 0\leqslant\rho\leqslant\infty$ and of lower order μ. A real number α is said to be admissible to $f(z)$ if $\alpha=0$ when $\rho=0, 0\leqslant\alpha<\rho$ when $0<\rho<\infty$ and $0<\alpha<\infty$ when $\rho=\infty$. We will use the usual symbols of the Nevanlinna theory of meromorphic functions: $T(r,f)$, $N(r,a,f), \delta(a,f)$ etc. (see [2]).

Now, we have introduced in [3] the following symbols in order to avoid the exceptional set in the second fundamental theorem of Nevanlinna for any admissible α to $f(z)$ and $r_0>0$:

$$T_\alpha(r,r_0,f)=\int_{r_0}^r T(t,f)/t^{1+\alpha}dt, \qquad N_\alpha(r,r_0,a,f)=\int_{r_0}^r N(t,a,f)/t^{1+\alpha}dt$$

and

$$\delta_\alpha(a,f)=1-\limsup_{r\to\infty}\frac{N_\alpha(r,r_0,a,f)}{T_\alpha(r,r_0,f)},$$

where a is any point on the Riemann sphere, and proved:

1) $T_\alpha(r,r_0,f)$ tends to the infinity monotonously as $r\to\infty$ and

$$\limsup_{r\to\infty}\frac{\log T_\alpha(r,r_0,f)}{\log r}=\begin{cases}\rho-\alpha & \text{for } \rho<\infty\\ \infty & \text{for } \rho=\infty,\end{cases}$$

$$\liminf_{r\to\infty}\frac{\log T_\alpha(r,r_0,f)}{\log r}\begin{cases}\geqslant\max(\mu-\alpha,0) & \text{for } \mu<\infty\\ =\infty & \text{for } \mu=\infty,\end{cases}$$

2) $\delta_\alpha(a,f)$ is independent of the choice of r_0 and for admissible $\beta(>\alpha)$ to $f(z)$

$$\delta(a,f)\leqslant\delta_\alpha(a,f)\leqslant\delta_\beta(a,f)\leqslant 1,$$

3) $\sum_a \delta_\alpha(a,f)\leqslant 2.$

We call $\delta_\alpha(a,f)$ α-deficiency of $f(z)$ at a. It is natural to consider whether the α-deficiency depends on the choice of origin or not as well as the Nevanlinna deficiency. In this note, we will show first that $\delta_\alpha(a,f)$ depends on the choice of origin by using Dugué's example ([1]) used for the case of $\delta(a,f)$, and next give some sufficient conditions under which $\delta_\alpha(a,f)$ is invariant under a change of origin by Valiron's method ([4]).

2. Example. Let

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