62. On the Asymptotic Distribution of Eigenvalues of Operators Associated with Strongly Elliptic Sesquilinear Forms

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1. **Introduction and main theorem.** The object of this note is to show that concerning the asymptotic distribution of eigenvalues of elliptic operators the results similar to those of S. Agmon [1], [2], R. Beals [3], etc. hold under somewhat different assumptions. Only an outline of the proof is presented here and the details will be published elsewhere.

Let Ω be a bounded domain of R^n having the restricted cone property ([2]). Let V be a closed subspace of $H_m(\Omega)$ containing $\mathring{H}_m(\Omega)$ and $a(u, v)$ be a symmetric integro-differential sesquilinear form of order m:

$$a(u, v)=\int_\Omega \sum_{|\alpha|, |\beta| \leq m} a_{\alpha\beta}(x) D^\alpha u \overline{D^\beta v}\, dx.$$

It is assumed that there exists a positive constant δ such that

$$a(u, u) \geqq \delta \|u\|_m^2 \qquad \text{for any } u \in V.$$

It is also assumed that $2m > n$. We denote by V^* the antidual of V. Then according to the usual convention we may consider $V \subset L^2(\Omega) \subset V^*$ algebraically and topologically. Let A be the operator associated with the sesquilinear form a:

$$a(u, v)=(Au, v) \qquad \text{for } u, v \in V,$$

where the bracket on the right denotes the pairing between V^* and V. A is a bounded linear operator on V onto V^*. For $x \in \Omega$ let $\delta(x) = \min\{1, \operatorname{dist}(x, \partial\Omega)\}$. We denote by $N(t)$ the number of eigenvalues of A which do not exceed $t > 0$.

Theorem. *Suppose that the coefficients of the highest order terms of a are Hoelder continuous of order h and other coefficients are bounded and measurable. Suppose also that*

$$\int_\Omega \delta(x)^{-p} dx < \infty$$

for some positive number $p<1$. Under the hypotheses stated above we have

(1) $$N(t)=c_0 t^{n/2m} + O(t^{(n-\theta)/2m})$$

as $t \to \infty$ where

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