86. *On a Certain Difference Scheme for Some Semilinear Diffusion System*

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Recently many mathematical models have been formulated by many physical mathematicians and mathematical physicists. Most of them are derived from the systems in physics, chemistry, medicine and ecology, for example, super conductivity system by T. Tsuneto and E. Abrahams [1], animal nerve axon system by A. L. Hodgkin and A. F. Huxley [2] and population dynamics system by Volterra and Kerner [3], etc. All of them are expressed by the following semilinear diffusion system:

(1) $$D_t U = A\Delta U + F(U),$$

where $U={}^t(u_1, u_2, \cdots, u_N)$, $F(U)={}^t(f_1(u_1, u_2, \cdots, u_N), f_2(u_1, u_2, \cdots, u_N), \cdots, f_N(u_1, u_2, \cdots, u_N))$, and A is a N-th order constant diagonal matrix with its non-negative element a_i for $i=1, 2, \cdots, N$.

Here we deal with the system (1) as an initial value problem in $H=\{x \in R_n, 0 \leqq t < +\infty\}$ with the initial data

(2) $$U(0, x)=\Phi(x),$$

where $\Phi={}^t(\phi_1, \phi_2, \cdots, \phi_N)$. Our concern is to formulate the stable difference scheme to resolve the problem (1) and (2).

First, we give the essential hypothesis to (1),

Hypothesis. (i) As for the matrix A,

$$a_{q_{m+1}} = a_{q_{m+2}} = \cdots = a_{q_{m+1}} = \bar{a}_{m+1},$$

where $m=0, 1, 2, \cdots, M-1$, $q_0=0$ and $q_M=N$.

(ii) For some constant vectors

$$D_m^p = (0, 0, \cdots, 0, d^p_{q_{m-1}+1}, d^p_{q_{m-1}+2}, \cdots, d^p_{q_m}, 0, \cdots, 0),$$

there exist non-negative constant c_m^p and non-negative $s_m^p(U)$ such that

$$D_m^p F(U) \leqq (c_m^p - D_m^p U) s_m^p(U)$$

for

$$U \in \bigcap_{m=1}^{M} \bigcap_{p=1}^{P(m)} \{U \in R^N, D_m^p U \leq c_m^p\},$$

where $m=1, 2, \cdots, M$ and $p=1, 2, \cdots, P(m)$.

We consider the following difference scheme to (1) and (2) under the Hypothesis,

(3) $$\frac{1}{\Delta t}(U^{i+1,J} - U^{i,J}) = \frac{1}{(\Delta x)^2} A \sum_{k=1}^{n} T_-^k T_+^k U^{i,J} + F(U^{i,J}) - S(U^{i,J})(U^{i+1,J} - U^{i,J})$$