83. *Some Radii Associated with Polyharmonic Equation. II*

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Introduction. In the preceding paper [2], we treated the polyharmonic inner radius of a domain and in the present paper we are going to deal with the polyharmonic outer radius. G. Pólya and G. Szegö [3] defined the outer radius of a bounded domain by a conformal correspondence from the exterior of a given bounded domain to that of a circle and showed that it can be also given by the Green's function of the exterior of a bounded domain relative to the Laplace's equation $\Delta u=0$. Moreover defining the biharmonic outer radius of a domain by the Green's function of the exterior of it concerning with the biharmonic equation $\Delta^2 u=0$, they calculated the ordinary outer and biharmonic outer radii of a nearly circular domain. The aim of this paper is to extend the above results. In the first place, we obtain the Green's function of the exterior of a disk with the pole the point at infinity relative to the n-harmonic equation $\Delta^n u=0$ and define the n-harmonic outer radius of a bounded domain. Applying the above results, we compute the n-harmonic outer radius of a nearly circular domain and it is noticeable that it is monotonously increasing with respect to integer n, which is contrary to the fact in case of inner radius.

1. Outer radii associated with polyharmonic equations.

We use the following notations hereafter. Let D be a bounded and simply connected domain in the complex z-plane, C the boundary of D, $\tilde{D}$ the exterior of D, $z=x+iy$ the variable point in $\tilde{D}$, r the distance from the origin to z and ∞ the point at infinity of the extended complex plane.

Definition 1. The function satisfying following two conditions is called the Green's function of D with the pole ∞ relative to the n-harmonic equation $\Delta^n u=0$.

(i) The function has in a neighbourhood of ∞ the form excepting plus and minus signs

$$\log r + ar^{2(n-1)} + P(x, y) + h_n(z),$$

where the function $P(x,y)$ is a polynomial of x and y with order $\leqq 2n-3$ and $h_n(z)$ satisfies the equation $\Delta^n u=0$ in $\tilde{D}$.

(ii) On the boundary C, the function and all its normal derivatives of order $\leqq n-1$ vanish.

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