100. *On Power Semigroups*

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1. If X is a semigroup, then the product of non-empty subsets of X can be defined in a natural way to produce a semigroup, which is called the power semigroup of X ([4]), and is denoted by $\mathfrak{X}(X)$. It is obvious that semigroups X and Y are isomorphic, then the power semigroups $\mathfrak{X}(X)$ and $\mathfrak{X}(Y)$ are isomorphic. This note is devoted to the converse question: if $\mathfrak{X}(X)$ and $\mathfrak{X}(Y)$ are isomorphic, must X and Y be isomorphic? We will answer this question for commutative semigroups whose ideals are all principal ideals. In the case for finite groups and chains, see [4].

2. By a partially ordered semigroup we mean a set X satisfying

(P1) X is a semigroup;

(P2) X is a partially ordered set under a relation $\leqq$;

(P3) $a \leqq b$ implies $ac \leqq bc$ and $ca \leqq cb$ for all $c \in X$ ([1], p. 153).

Let X and Y be partially ordered semigroups. By an *o*-isomorphism of X onto Y we mean a one-to-one mapping $\emptyset$ of X onto Y such that

(O1) $\emptyset(ab) = \emptyset(a)\emptyset(b)$ for all $a, b \in X$;

(O2) $a \leqq b$ in X if and only if $\emptyset(a) \leqq \emptyset(b)$ in Y.

3. Let X be a semigroup and $\mathfrak{X}(X)$ the set of all non-empty subsets of X. A binary operation is defined in $\mathfrak{X}(X)$ as follows: For A, $B \in \mathfrak{X}(X)$

$$AB = \{ab\,;\, a \in A, b \in B\}.$$

Then it is well-known and is easily seen that $\mathfrak{X}(X)$ is a semigroup. This semigroup $\mathfrak{X}(X)$ is called the power semigroup of X.

We define a relation $\leqq$ on $\mathfrak{X}(X)$ as follows; For $A, B \in \mathfrak{X}(X)$,

$$A \leqq B \text{ if and only if } A \subseteq B.$$

Then, as is well-known ([2], p. 132), $\mathfrak{X}(X)$ is a partially ordered set under this relation $\leqq$ satisfying the condition (P3), that is, $\mathfrak{X}(X)$ is a partially ordered semigroup.

Let $\mathfrak{I}(X)$ be the set of all ideals of a semigroup X and $\mathfrak{P}(X)$ the set of all principal ideals of X. Then clearly $\mathfrak{I}(X)$ is a subsemigroup of $\mathfrak{X}(X)$.

4. Proposition 1. *Let $\emptyset$ be an o-isomorphism of $\mathfrak{X}(X)$ onto $\mathfrak{X}(Y)$ and $\emptyset^*$ the restriction of $\emptyset$ on $\mathfrak{I}(X)$. Then $\emptyset^*$ maps $\mathfrak{I}(X)$ onto $\mathfrak{I}(Y)$.*