167. On the Generalized Decomposition Numbers of the Alternating Group

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(Comm. by Kenjiro SHODA, M. J. A., Dec. 13, 1971)

The generalized decomposition numbers of the symmetric group are rational integers ([5], [13]), but those of the alternating group are not necessarily rational integers ([5]). The main purpose of this paper is to give a proof of the following theorem ([4]).

Theorem 1. *The generalized decomposition numbers of the alternating group for $p=2$ are rational integers.*

Throughout this paper, we consider the representations of groups over the algebraically closed field of characteristic 2. Let x be a 2-element of the alternating group A_n, and let $N_A(x)$ be the normalizer of x in A_n. In section 2 we shall prove that every 2-block B_σ^* of $N_A(x)$ is characterized by a 2-core $[\alpha_0]$, and then B_σ^* with the 2-core $[\alpha_0]$ determines the 2-block B_σ of A_n with the same 2-core $[\alpha_0]$.

1. The generalized symmetric group $S(a_i, 2^i)$ is the semi-direct product of the normal subgroup Q_i of order $(2^i)^{a_i}$ and the subgroup $S_{a_i}^*$ which is isomorphic to the symmetric group S_{a_i}([9]):

(1.1) $$S(a_i, 2^i)=S_{a_i}^* Q_i, \quad S_{a_i}^* \cap Q_i=1, \quad S_{a_i}^* \cong S_{a_i}.$$

Evidently we have $S(a_0, 1)=S_{a_0}$. Since $S(a_i, 2^i)/Q_i \cong S_{a_i}$, we see that every modular irreducible character of $S(a_i, 2^i)$ is given by the modular irreducible character of S_{a_i}.

Let G be a subgroup of the symmetric group S_n and let us denote by G^+ the subgroup $G \cap A_n$ of G. Then we have $G=G^+$ or $(G: G^+)=2$. Since $(Q_i: Q_i^+)=2$ for $i>0$, we see that

(1.2) $$S(a_i, 2^i)^+=S_{a_i}^* Q_i^+.$$

Let y be an arbitrary 2-regular element of $S(a_i, 2^i)$. Then y is the even permutation and hence $y \in S(a_i, 2^i)^+$. It follows from $S(a_i, 2^i)^+/Q_i^+ \cong S_{a_i}$ that every representation of $S(a_i, 2^i)^+$ obtained by restricting the modular irreducible representation of $S(a_i, 2^i)$ remains irreducible. If we denote by φ_κ^i $(\kappa=1, 2, \cdots, m_i)$ the modular irreducible characters of S_{a_i}, then the modular irreducible characters of $S(a_i, 2^i)$ and $S(a_i, 2^i)^+$ are also given by $\varphi_\kappa^i(y)$. This implies that the representation $\tilde{U}_\kappa^i$ of $S(a_i, 2^i)$ induced from the indecomposable constituent U_κ^i of the regular representation of $S(a_i, 2^i)^+$ is the indecomposable constituent of the regular representation of $S(a_i, 2^i)$ ([8]) and hence if we denote by $\tilde{c}_{\kappa\lambda}$ and