5. *Results Related to Closed Images of M-Spaces. III*

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(Comm. by Kinjirô KUNUGI, M. J. A., Jan. 12, 1972)

1. Introduction. Throughout this paper by a space we shall mean a T_1-space, and by N the set of natural numbers. For a space X let us consider the following conditions, where the same terminology as in [9] will be used.

(CM): There exists a sequence $\{\mathfrak{F}_n \mid n \in N\}$ of hereditarily closure-preserving closed covers of X such that

(i) any sequence $\{A_n\}$ with $x \in A_n \in \mathfrak{F}_n$ for $n \in N$ is either hereditarily closure-preserving or a q-sequence at a point x of X, and

(ii) every point x of X has a q-sequence $\{A_n\}$ with $x \in A_n \in \mathfrak{F}_n$ for $n \in N$.

(qk) X is a quasi-k-space (Nagata [11]).

(q) X is a q-space in the sense that each point of X has a q-sequence which consists of neighborhoods of x (Michael [5]).

(sst) X is semi-stratifiable (cf. Creede [2]).

(σ) X is a σ-space in the sense that there is a σ-locally finite network for X (Okuyama [13]).

As is known, (σ) implies (sst) and (q) implies (qk) if X is regular (cf. [6, Theorem 2. F. 2]), but (q) does not imply (qk) if X is Hausdorff (cf. [6, Example 10. 11]).

The purpose of this paper is to prove the following theorems except Theorem 1.1 which was obtained in [9] and is stated here for comparison.

Theorem 1.1. *A regular space X is the closed image of a regular M-space iff* (CM) *and* (qk) *hold.*

Theorem 1.2. *A Hausdorff space X is the closed image of a metric space iff* (CM), (qk) *and* (sst) (*or* (σ)) *hold.*

Theorem 1.3. *A Hausdorff space X is metrizable iff* (CM), (q) *and* (sst) (*or* (σ)) *hold.*

Theorem 1.4. *A space X is an M^*-space iff* (CM) *and* (q) *hold.*

Theorem 1.5. *A regular space X is semi-metrizable iff* (q) *and* (sst) *hold.*

In view of Theorem 1.4, Theorem 1.3 implies Theorem 1.6 below, which is due to Ishii and Shiraki [4] for (sst) and to Shiraki [15] for (σ),[1] but we shall first give a new proof of the latter and then make

1) I have heard from J. Nagata that F. Slaughter proved that a Hausdorff space is metrizable iff it is an M-space and a σ-space.