30. A Representation of Entropy Preserving Isomorphisms between Lattices of Finite Partitions*)

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1. Introduction. We showed in [2] that the entropy in the information theory can be characterized as the semivaluation on the semilattice, and we discussed about measure preserving transformations as entropy preserving lattice-isomorphisms on the space of all measurable finite partitions. In this paper, we shall analyse the relation between measure preserving transformations and entropy preserving lattice-isomorphisms more minutely. Considering an arbitrary entropy preserving lattice-isomorphism which is defined abstractly as a mapping from the family of all finite partitions of a probability measure space onto that of another probability space, we shall see that such a lattice-isomorphism induces an isometrical isomorphism from the measure algebra of the former space onto the algebra of the latter. Hence, on some natural measure spaces, entropy preserving lattice-isomorphisms are represented as measure preserving point transformations. And we shall see that two concepts of conjugacy (cf. Billingsley [1], p. 66) and isomorphism of AD-systems (cf. [2]) are equivalent for the general dynamical systems.

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2. Notations and definitions. In what follows we deal with two probability measure spaces $(X, \mathcal{X}, p)$ and $(Y, \mathcal{Y}, q)$. When we indicate either $(X, \mathcal{X}, p)$ or $(Y, \mathcal{Y}, q)$, we represent it commonly by $(Z, \mathcal{Z}, r)$. The quotient algebra $\mathcal{Z}/\mathcal{N}$, where $\mathcal{N}$ is the ideal of null sets, is simply written by $\tilde{\mathcal{Z}}$, and called a measure algebra with the measure r. The sets in $\mathcal{Z}$ are denoted by $A, B, C, \cdots$ and the elements in $\tilde{\mathcal{Z}}$ are denoted by $\tilde{A}, \tilde{B}, \tilde{C}, \cdots$, where $\tilde{A}$ represents the residue class containning the set A in $\mathcal{Z}$. The class of all finite measurable partitions of Z, which is a lattice with the order of refinement $\prec$ (and $\vee$, $\wedge$ for the two operations of join and meet for the lattice), is denoted by F_Z, and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}, \cdots$ are elements in F_Z. The entropy function $H(\cdot)$ on F_Z is defined by

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