23. Approximate Propervalues and Characters of C^*-algebra

By Isamu KASAHARA*) and Hiroshi TAKAI**)

(Comm. by Kinjirô KUNUGI, M. J. A., Feb. 12, 1972)

1. Introduction. Recently, Bunce [2] established a kind of reciprocity among the characters of singly generated C^*-algebras and approximate propervalues of the generators. He proved, among others, the following theorem:

Theorem 1. *If A is a hyponormal operator acting on a Hilbert space $\mathfrak{H}$ and λ is an approximate propervalue of A, then there is a character ϕ on the C^*-algebra $\mathfrak{A}$ generated by A (and 1) such that*

$$\phi(A)=\lambda. \tag{1}$$

In the above theorem, a *character* means a multiplicative state of $\mathfrak{A}$. A *state* ϕ is a positive linear functional on $\mathfrak{A}$ with $\phi(1)=1$, and ϕ is *multiplicative* if

$$\phi(AB)=\phi(A)\phi(B), \tag{2}$$

for every $A, B \in \mathfrak{A}$.

Bunce [2] also proved the following theorem which is originally established by Arveson:

Theorem 2. *If λ is a spectre of A with $|\lambda|=\|A\|$, then there is a character ϕ on $\mathfrak{A}$ which satisfies* (1).

In the present note, we shall show that a kind of approximate propervalues has a closed connection with the characters of singly generated C^*-algebras. As consequences, the above mentioned theorems of Arveson and Bunce are proved under a unified method.

2. Normal approximate propervalues. In this note, we shall mean an operator A is a bounded linear operator acting on $\mathfrak{H}$. Following after Halmos [4], we shall call a complex number λ is an *approximate propervalue* of A *provided that λ and A* satisfy

$$\|Ax_n-\lambda x_n\|\to 0 \qquad (n\to\infty) \tag{3}$$

for a sequence $\{x_n\}$ of unit vectors. Furthermore, if λ and A satisfy (3) and

$$\|A^*x_n-\lambda^*x_n\|\to 0 \qquad (n\to\infty), \tag{3*}$$

then λ is called a *normal approximate propervalue* of A (in [7], λ is called an approximate *reducing* propervalue). By the spectral theorem, we can see that every spectre of a normal operator is a normal approximate propervalue.

*) Momodani Senior Highschool, Osaka.
**) Department of Mathematics, Osaka Kyoiku University.