21. On the Topological Spaces with the $\mathfrak{B}$-property

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Recently, P. Zenor [9] defined the topological class contained in the countably paracompact spaces. It is the generalization of C. H. Dowker ([1], Theorem 2) or F. Isikawa [2]. On the other hand, S. Sasada [7] defined the α_i-spaces ($i=1,2$) in addition the normality (normal $\mathfrak{B}$-spaces are α_2-spaces).

The purpose of this paper is to study some characterizations and properties of $\mathfrak{B}$-spaces. F. Isikawa [2] proved the following theorem:

Theorem 1. *In order that a topological space be countably paracompact, it is necessary and sufficient that if a decreasing sequence* $\{F_i \mid i=1,2,\cdots\}$ *of closed sets with vacuous intersection is given, then there exists a decreasing sequence* $\{G_i \mid i=1,2,\cdots\}$ *of open sets such that* $\{\overline{G_i} \mid i=1,2,\cdots\}$ *has a vacuous intersection and* $G_i \supset F_i$ *for* $i=1,2,\cdots$.

At this time, we can naturally define the $\mathfrak{B}$-space, that is, a topological space X is said to be a $\mathfrak{B}$-*space* if every monotone decreasing[1] family $\{F_\alpha \mid \alpha \in A\}$ of closed sets with the vacuous intersection has the monotone decreasing family $\{G_\alpha \mid \alpha \in A\}$ of open sets such that $\bigcap_{\alpha \in A} \overline{G_\alpha} = \emptyset$ and $G_\alpha \supset F_\alpha$ for each $\alpha \in A$. From the above definition, the $\mathfrak{B}$-property is weakly hereditary[2] and the following is trivial:

Proposition. *In order that a topological space X be a $\mathfrak{B}$-space, it is necessary and sufficient that every monotone increasing*[1] *open covering* $\{G_\alpha \mid \alpha < \lambda\}$ *of X has the monotone increasing open covering* $\{U_\alpha \mid \alpha < \lambda\}$ *of X such that* $G_\alpha \supset \overline{U_\alpha}$ *for each* $\alpha < \lambda$.

In order to prove some theorems, we shall use the following:

Lemma. *Let X be a topological space, then X is countably paracompact if and only if every monotone increasing countable open covering* $\mathfrak{U}$ *of X has the σ-cushioned*[3] *open refinement.*

The proof of this lemma is easily seen from Theorem 1.

Theorem 2. *In a topological space X, the following properties are equivalent:*

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1) A family $\{F_\alpha \mid \alpha \in A\}$ of subsets of X is *monotone increasing* (resp. *monotone decreasing*) if A is well ordered and $F_\alpha \supset F_\beta$ (resp. $F_\alpha \subset F_\beta$) for each $\alpha \geq \beta$; $\alpha, \beta \in A$.

2) A topological property P is said to be *weakly hereditary* if every closed subspace of X has the property P whenever X has the property P.

3) See E. Michael [4].