15. *Some Investigations on Many Valued Logics*

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In their book [1], Chang and Keisler developed a study of theories of models with truth values in compact Hausdorff spaces. One of the main reasons why they required some topological properties is that a basic tool used there is the compactness theorem. But, we can develop a study of model theories on logics without compactness properties.

In this paper, we shall study logics with truth values in some set X, in which we don't assume any topological properties and some theories of models on these logics by the method developed in [2]–[4].

Many valued logic $\mathcal{L}_1=\mathcal{L}_1(X, C, Q, \underset{\sim}{1})$. Let X be a non empty *countable* set with a designated element $\underset{\sim}{1} \in X$, X^* be the set of all non empty subsets of X, C be a set of finitary functions on X, and Q be a set of unary functions on X^* to X. Then by the usual manner, we can construct a many valued logic $\mathcal{L}_1=\mathcal{L}_1(X, C, Q, \underset{\sim}{1})$ with equality $\simeq$ except that we admit the following role of quantifiers; if $q \in Q$ and Σ is a set of formulas in $\mathcal{L}_1$ such that $1 \leqq \overline{\overline{\Sigma}} \leqq \overline{\overline{X}}$, then $q(\Sigma)$ is a formula in $\mathcal{L}_1$. Also, we can define the semantical notions such as $\mathcal{L}_1$-structure $\mathcal{A}$, and assignment r in $\mathcal{A}$, $\sigma[\mathcal{A}, r] \in X$ for any formula σ in $\mathcal{L}_1$.

If σ and τ are formulas in $\mathcal{L}_1$, "σ is a consequence of τ" (written by $\tau \models \sigma$) means that $\tau[\mathcal{A}, r]=\underset{\sim}{1}$ implies $\sigma[\mathcal{A}, r]=\underset{\sim}{1}$ for any $\mathcal{A}, r$ and "σ is valid" (written by $\models \sigma$) means that $\sigma[\mathcal{A}, r]=\underset{\sim}{1}$ for any $\mathcal{A}, r$.

Two valued logic $\mathcal{L}=\mathcal{L}(\mathcal{L}_1)$ as a metalogic of $\mathcal{L}_1$. $\mathcal{L}=\mathcal{L}(\mathcal{L}_1)$ can be defined from $\mathcal{L}_1$ by the following rules:

(1) If σ is a formula in $\mathcal{L}_1$ and $x \in X$, then (σ, x) is formula in $\mathcal{L}$. (if σ is an atomic formula in $\mathcal{L}_1, (\sigma, x)$ is called an *atomic* formula in $\mathcal{L}$).

(2) Usual closure under two valued logical operations $\neg, \wedge, \vee, \forall, \exists$ except that $\wedge$ and $\vee$ are only applied to sets Φ of formulas such that $1 \leqq \overline{\overline{\Phi}} \leqq \overline{\overline{X}}$.

If a formula θ in $\mathcal{L}$ can be constructed from only atomic formulas in $\mathcal{L}$ by applying $\neg, \wedge, \vee, \forall, \exists$, then θ is called *normal*. For any $\mathcal{L}_1$-structure $\mathcal{A}$, any assignment r in $\mathcal{A}$ and any formula θ in $\mathcal{L}$, we can define the satisfaction relation $\mathcal{A} \models \theta[r]$ by the usual method. Let $FM(\mathcal{L})$ and $PFM(\mathcal{L})$ be the set of formulas in $\mathcal{L}$ and the set of valid formulas in $\mathcal{L}$. Then clearly these $FM(\mathcal{L})$ and $PFM(\mathcal{L})$ satisfy the conditions stated in [2].