13. *On Deformations of Holomorphic Maps*

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(Comm. by Kunihiko KODAIRA, M. J. A., Feb. 12, 1972)

0. Introduction. The modern deformation theory has begun with the splendid work of Kodaira-Spencer [1] followed by [2], [3]. Moreover Kodaira has investigated families of submanifolds of a fixed compact complex manifold in [4]. The next natural problem is to investigate "deformations of holomorphic maps". I intend to give here a statement of fundamental results and some applications. Details will be published elsewhere.

1. Notations and conventions. We denote by X, Y, Z compact complex manifolds and by $p: \mathcal{X} \to M$, $q: \mathcal{Y} \to N$, $\pi: \mathcal{Z} \to S$ complex analytic families of compact complex manifolds (see [1] for the definition).

We say that two holomorphic maps $f: X \to Y$ and $f': X' \to Y$ are equivalent if there exists a complex analytic isomorphism $h: X \to X'$ such that $f = f' \circ h$.

2. Deformations of non-degenerate holomorphic maps. By a family of holomorphic maps into a fixed compact complex manifold Y, we mean a quadruplet $(\mathcal{X}, \Phi, p, M)$ of complex analytic family $p: \mathcal{X} \to M$ and a holomorphic map $\Phi: \mathcal{X} \to \mathcal{Y} = Y \times M$ over M in the sense that $p = pr_2 \circ \Phi$.

We define the concept of completeness of a family of holomorphic maps into Y as in the theory of deformations of compact complex manifolds [1].

Let $(\mathcal{X}, \Phi, p, M)$ be a family of holomorphic maps into Y, $0 \in M$, $X = X_0 = p^{-1}(0)$ and let $f = \Phi_0: X \to Y$ be the induced holomorphic map. Then we have an exact sequence of sheaves on X:

$$\Theta_X \xrightarrow{F} f^*\Theta_Y \xrightarrow{P} \mathcal{T} \longrightarrow 0$$

where θ denotes the sheaf of germs of holomorphic vector fields, $\mathcal{T} = \mathcal{T}_{X/Y}$ is the cokernel of the canonical homomorphism F and P is the natural projection.

For simplicity we assume that f is non-degenerate (i.e. $\operatorname{rank}_z df = \dim X$ for some point $z \in X$). Then the homomorphism F is injective. If f is an embedding, $\mathcal{T}$ is nothing but the normal bundle $\mathcal{N}$.

Now we define a characteristic map

$$\tau = \tau_0: T_0(M) \longrightarrow H^0(X, \mathcal{T})$$

($T_0(M)$ is the tangent space of M at 0) by the formula