12. *The Stable Jet Range of Differential Complexes*

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1. Let M be an n-dimensional smooth manifold with countable basis. A topological space W is called an inductive vector bundle over M if there is an increasing sequence of finite-dimensional smooth vector bundles W_k ($k=0,1,\cdots$) over M, W_k being a subbundle of W_{k+1}, such that $\lim \dim W_k=\infty$ and $W=\varinjlim W_k$ (inductive limit space). Then W becomes a fibre space over M. We can naturally define the space of smooth cross-sections $\Gamma(W)$ which has a module structure over the algebra $\mathcal{E}$ of smooth functions on M. We endow $\Gamma(W)$ with a nuclear topology such that, if M is compact, $\Gamma(W)$ coincides with the inductive limit space $\varinjlim \Gamma(W_k)$ where each $\Gamma(W_k)$ is assumed to have the C^∞-topology. Two inductive vector bundles W and W' are called isomorphic if $\Gamma(W)\cong\Gamma(W')$ as topological vector spaces and $\mathcal{E}$-modules.

We say that a sequence

$$0 \longrightarrow \textstyle\sum^0 \xrightarrow{d} \sum^1 \xrightarrow{d} \sum^2 \xrightarrow{d} \cdots$$

is a differential complex over M if i) each $\sum^p$ is an $\mathcal{E}$-submodule of some $\Gamma(W^p)$, ii) d is continuous and $d\circ d=0$, iii) $\operatorname{supp} dL\subset \operatorname{supp} L$ where $\operatorname{supp} L$ means the support of $L\in \sum^p$.

2. Suppose that finite-dimensional smooth vector bundles E and F over M be given. Note that the jet bundles $J^k(E)$ of E ($k=0,1,2,\cdots$) have the canonical surjective maps $\lambda^k: J^{k+1}(E)\to J^k(E)$. Hence we obtain the injective maps

$$(\lambda^k)^*: \operatorname{Hom}(J^k(E),F)\to \operatorname{Hom}(J^{k+1}(E),F)$$

($k=0,1,2,\cdots$), and thus the inductive vector bundle

$$C^1(E,F)=\varinjlim \operatorname{Hom}(J^k(E),F)$$

is constructed. The cross-section space of $C^1(E,F)$ is regarded as the space of the differential operators from $\Gamma(E)$ to $\Gamma(F)$.

More generally, set

$$C^p(E,F)=\varinjlim \operatorname{Hom}(\wedge^p J^k(E),F), \qquad p=1,2,\cdots$$
$$C^0(E,F)=F,$$

and write $C^p[E,F]=\Gamma(C^p(E,F))$ for $p=0,1,\cdots$.

Proposition. *Each $C^p[E,F]$ is canonically identified with the space of continuous multilinear alternating mappings from $\Gamma(E)\times\cdots\times\Gamma(E)$ (p times) to $\Gamma(F)$ satisfying the condition*

$$\operatorname{supp} L(\xi_1,\cdots,\xi_p)\subset \operatorname{supp}\xi_1\cap\cdots\cap\operatorname{supp}\xi_p.$$

3. Our main concern is to study the cohomological structure of a