11. On Some Subgroups of the Group $PSp(2n, q)$

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Introduction. We say that a subgroup H of a group G is of rank 2 if the number of double cosets $H\backslash G/H$ is equal to 2. Any subgroup of rank 2 of G is the stabilizer of a point of some doubly transitive permutation representation of G, and vice versa.

In [2] the author has determined the rank 2 subgroups of the symplectic group $Sp(2n, 2)$. After that, a remarkable paper of G. M. Seitz [7] has been appeared, which determined all flag transitive subgroups of the finite Chevalley groups (among which all simple groups of Lie type are included) and which asserts together with the result of J. A. Green [5] that there exists an integer N (which depends only on the type of the associated Weyl group) such that if $q \geqslant N$ then rank 2 subgroups of the Chevalley group of the type defined over the finite field $GF(q)$ are among the maximal parabolic subgroups. However, any effective bound for the integer N is not obtained at present.

The purpose of this note is to give an outline of the proof of the following Theorem 1. The proof is done combining the idea used in [2] and the main result given in Seitz [7]. Details will be published elsewhere together with that of [2].

Theorem 1. *The simple group $PSp(2n, q)$, $n \geqslant 10$ and $q > 2$, has no subgroup of rank* 2.

Remark. The assumption that $n \geqslant 10$ is not a serious one but a mere convention not to make the argument so complicated, and it is possible to loosen the restriction a little, say up to $n=7$ (or 6). Nevertheless for small values of n some additional special treatments are needed, and it is not yet done completely at the time of writing this note. However, it will be not so difficult to settle those remaining cases. Our method given here is also applicable for *any* finite Chevalley groups with high rank (as BN-pair)(it is sufficient if we take $n \geqslant 15$, say). These will be treated in a subsequent paper.

§1. The group $Sp(2n, q)$. We may define $G = Sp(2n, q)$, the symplectic group defined over the finite field $GF(q)$, by

$$G = \left\{ X \in GL(n, q) \,;\, {}^tXJX = J, \text{ with } J = \begin{pmatrix} & -I_n \\ I_n & \end{pmatrix} \right\}.$$

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