40. *Remarks on the Conductor of an Elliptic Curve*

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1. In this note we treat, in a sense, a generalization of the results by Ogg [2] on the conductor of an elliptic curve defined over the field of rational numbers.

2. Extending the diophantine lemma of Ogg [2], we get

Proposition. *For a given odd prime p such that* $p\equiv 3$ *or* $5 \pmod 8$

i) *All the integer solutions of the equation* $X^2-1=2^\alpha p^\beta$ *are, including the trivial case* $\alpha\beta=0$, $(|X|, 2^\alpha p^\beta)=(2,3), (3,2^3), (5,2^3 3), (7,2^4 3), (9,2^4 5)$ *and* $(17, 2^5 3^2)$.

ii) *All the integer solutions of the equation* $X^2+1=2^\alpha p^\beta$ *are trivially* $|X|=1$ *if* $p\equiv 3 \pmod 8)$ *and are given by* $\alpha=0, \beta=1$ *and* $\alpha=1, \beta=1, 2$ *or* 4 *if* $p\equiv 5 \pmod 8$. *Especially we have* $\beta=4$ *if and only if* $p=13$, $X=239$.

iii) *The equation* $2X^2-1=p^\alpha (\alpha>0)$ *has no integer solution.*

iv) *All the integer solutions of the equation* $2X^2+1=p^\alpha (\alpha>0)$ *are given by* $\alpha=1$ *or* 2 *and* $(|X|, p^\alpha)=(11, 3^5)$ *if* $p\equiv 3 \pmod 8$ *and none if* $p\equiv 5 \pmod 8$.

v) *We assume that here p satisfies the conjecture of Ankeny-Artin-Chowla and the analogy* ([1] Chap. 8). *Then all the integer solutions of the equation* $|\pm p^\alpha - X^2|=2^\beta$ *are, except trivial solutions* $(\pm p^\alpha, |X|)=(1,3)$ *and* $(-1,1)$, *given by* $\alpha=\beta=1$; $(\pm p^\alpha, |X|)=(3^2,1), (3^2,5), (3^4,7), (3,2), (-3,1)$ *and* $(3^3,5)$ *if* $p\equiv 3 \pmod 8$, $\alpha=1, \beta=0$; $\alpha=1, \beta=2$; $(\pm p^\alpha, |X|)=(5^2,3)$ *and* $(5^3,11)$ *if* $p\equiv 5 \pmod 8$.

vi) *All the integer solutions of the equation* $pX^2-Y=\pm 2^\alpha$, $Y=\pm 2^\beta$ *are either* $2|X, 4|Y$ *or* $(|X|, Y)=(1,4), (1,1), (1,2)$ *and* $(1,-1)$ *if* $p=3$, $(1,4)$ *and* $(1,1)$ *if* $p=5$ *and none if* $p\neq 3, 5$.

3. For a given positive integer N, it is difficult in general to determine all the elliptic curves with the conductor N. This may be treated as problems in diophantine equations. However, when the curve is of special form, that is, when it has a rational point of order 2, we obtain the following theorems by using the above proposition.

Theorem 1. *All the elliptic curves with the coductor* $N=2^m p^n$ *(where* $p\equiv 3$ *or* $5 \pmod 8$, $p\neq 3$; *m and n are positive integers) that have a rational point of order* 2 *are effectively determined under the truth of the conjecture of Ankeny-Artin-Chowla and the analogy. Particularly if* $p-2$ *or* $p-4$ *is a square number, then the assumption on the con-*