52. The Theory of Nuclear Spaces Treated by the Method of Ranked Space. VI

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In the paper [8], we have studied the dual space of the extended nuclear space. In this paper we shall continue to do it.

§ 7. The dual space. (2).

Lemma 39. (1) *$V^*(0, h, i)$ is circled.*

(2) *$V^*(0, h, i)+V^*(0, k, j)=V^*(0, [hk/h+k], \min(i, j))$ for $h, k>1$.*

Proof. (1) It is clear.

(2) Suppose $i<j$. Then we have

$$V^*(0, h, i)+V^*(0, k, j)\subseteq V^*(0, h, j)+V^*(0, k, j)$$

by Lemma 37 in [8]. Now, let F_1 and F_2 belong to $V^*(0, h, j)$ and $V^*(0, k, j)$, respectively. Then we have $|F_1(g)|<\varepsilon_j/h$ and $|F_2(g)|<\varepsilon_j/k$ to every $g\in \hat{V}_j(0, 1, j)$, hence we obtain $|F_1(g)+F_2(g)|\leqq|F_1(g)|+|F_2(g)| <\varepsilon_j(h+k)/hk<\varepsilon_j/l$, where $l=[hk/h+k]$. This proof is complete. The sequence of neighbourhoods, $\{V^*(0, \gamma(h), i(h))\}$, where

$$V^*(0, \gamma(h), i(h))\supseteq V^*(0, \gamma(h+1), i(h+1)), \gamma(h)\leqq\gamma(h+1)$$

and $\gamma(h)\to\infty$ as $h\to\infty$, is a fundamental sequence of neighbourhoods in $\hat{\Phi}'$.

Lemma 40. *If $\{V^*(0, \gamma(h), i(h))\}$ is a fundamental sequence of neighbourhoods in $\hat{\Phi}'$, then $F\in V^*(0, \gamma(h), i(h))$ for every integer h implies $F=0$, that is, $F(g)=0$ for every $g\in\hat{\Phi}$.*

Proof. By Lemma 38 in [8], we have $\min_h\{i(h)\}\geqq 1$. We write briefly $\min_h\{i(h)\}=j$. Hence there exists some integer N such that the relation $h\geqq N$ implies $i(h)=j$. The fact that F belongs to $V^*(0, \gamma(h), j)$ for $h\geqq N$ follows $F\in M_j^0$ and $|F(g)|<\varepsilon_j/\gamma(h)$ for $g\in\hat{V}_j(0, 1, j)$. And since $g/2\hat{P}_j(g)$ belongs to $\hat{V}_j(0, 1, j)$ for any element $g\in\hat{\Phi}$ with $P_j(g)\neq 0$, we see $|F(g)/2\hat{P}_j(g)|<\varepsilon_j/\gamma(h)$. Consequently we obtain

$$|F(g)|<2\varepsilon_j\hat{P}_j(g)/\gamma(h).$$

That shows $F(g)=0$ for every $g\in\hat{\Phi}$. This proof is complete.

Now, we can prove that the linear space $\hat{\Phi}'$ is a linear ranked space, by M. Washihara, [3].

Theorem 7. *The linear ranked space $\hat{\Phi}'$ is complete with respect to the R-convergence.*

Proof. Let $\{F_n\}$ be an R-cauchy sequence of elements in $\hat{\Phi}'$. Then there exists some fundamental sequence of neighbourhoods

$$\{V^*(0, \gamma(h), i(h))\}$$