69. A Note on the Dilation Theorems. II

By Masahiro NAKAMURA and Hiroshi TAKAI
Department of Mathematics, Osaka Kyoiku University

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1. In the previous note [9], one of the authors discussed, jointly with Yamada, the mutual dependency of several dilation theorems. Especially, it is pointed out that Stinespring-Umegaki's algebra dilation theorem implies the so-called strong dilation theorem of Sz.-Nagy. However, the proofs of the implication are somewhat lengthy. In the present note, it will be shown that Stinespring-Umegaki's theorem can serve a proof of more general dilation theorem of Foiaş-Suciu [2]. Some consequences are also discussed.

2. The following theorem is the algebra dilation theorem due to [7] and [10]:

Theorem 1 (Stinespring-Umegaki). *If V is a completely positive (or positive definite) linear mapping defined on a unital C^*-algebra B with the range in the algebra $B(H)$ of all operators on a Hilbert space H, and V satisfies $V1=1$, then there is a ($*$-preserving) representation U of B on K such that*

(1) $$Vf=pUf|H$$

for any $f \in B$, where K includes H as a subspace and p is the projection of K onto H.

In the present note, the notion of the complete positivity is not necessary, since Stinespring [7; Theorem 4] established that the complete positivity coincides with the usual positivity if B is commutative which is the case treated in this note. Exactly, in the present note, B is always the algebra $C(X)$ of all continuous functions defined on a compact Hausdorff space X equipped with the sup-norm.

3. A subalgebra A of $C(X)$ is a *function algebra* on X if A satisfies

(i) A contains the constants, and

(ii) A separates the points of X.

A function algebra A is a *Dirichlet algebra* on X if the real part $\mathrm{Re}\,A$ of all real parts of functions belonging to A is dense in the algebra of all real continuous functions on X.

An *operator representation* V of a function algebra A on a Hilbert space H is an algebra homomorphism of A into $B(H)$ which satisfies

(2) $$V1=1$$