62. *Local Boundedness of Monotone-type Operators*[*)]

By P. M. FITZPATRICK,[**)] P. HESS,[***)] and Tosio KATO[***)]

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In this note we give a simple proof that certain monotone-type operators are locally bounded in the interior of their domains, thus generalizing a result of [1]. As special cases, we obtain the local boundedness for monotone operators from a Fréchet space to its dual and for accretive operators in a Banach space with a uniformly convex dual.

In what follows let X, Y be metrizable linear topological spaces. Further assume that Y is locally convex and complete (Fréchet space). We denote by $\langle\ ,\ \rangle$ the pairing between Y and its dual Y^*. We introduce a metric in X and denote by B_r the open ball in X with center 0 and radius $r>0$.

Let T be a mapping of X into 2^{Y^*}, with domain $D(T)=\{x \in X : Tx \neq \emptyset\}$ and graph $G(T)=\{(x, f) \in X \times Y^* ; f \in Tx\}$. Let F be a function on X to Y. Slightly generalizing a definition used in [1], we say T is F-monotone if $\langle F(x_1-x_2), f_1-f_2\rangle \geq 0$ for $(x_j, f_j) \in G(T), j=1, 2$.

Theorem. *Assume that there is $r_0>0$ such that*

(i) *F is uniformly coutinuous on B_{r_0} to Y.*

(ii) *For each $r<r_0, F(B_r)$ is absorbing in Y.*

(iii) *For each $u \in X$, the set $\{F(z-u)-Fz ; z \in B_{r_0}\}$ is bounded in Y.*

If $T: X \to 2^{Y^*}$ is F-monotone, then T is locally bounded at each interior point x_0 of $D(T)$, in the following sense: for each sequence $\{(x_n, f_n)\}$ in $G(T)$ with $x_n \to x_0$, $\{f_n\}$ is equicontinuous.

Examples. 1. Let $Y=X$ and $F=$identity map in X. Then F-monotonicity means monotonicity in the sense of Minty-Browder. Conditions (i) to (iii) are trivially satisfied, and the theorem shows that a monotone operator from a Fréchet space X to X^* is locally bounded in the interior of its domain (cf. [2], [3]).

2. Let X be a Banach space with X^* uniformly convex, and let $Y=X^*$ so that $Y^*=X^{**}=X$. Let F be the (normalized) duality map of X to X^*. Then F-monotonicity means accretiveness in the usual sense. It is known that F is onto X^* and uniformly continuous on any bounded set in X. Thus (i) to (iii) are satisfied, and the theorem shows

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**) Courant Institute of Math. Sciences, New York University, U. S. A.

***) Department of Math., University of California, Berkeley, U. S. A.