101. On Complex Parallelisable Manifolds and their Small Deformations

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0°. Introduction. By a complex parallelisable manifold we mean a compact complex manifold with the trivial holomorphic tangent bundle. Wang [7] showed that a complex parallelisable manifold is the quotient space of a simply connected complex Lie group by one of its discreet subgroups.

This note is a preliminary report on our recent results on complex parallelisable manifolds and their small deformations. Details will appear in the forthcoming paper [5].

1°. Let X be a compact complex manifold of $\dim n$. We denote by $\mathcal{O}$ and Ω^p the sheaf of germs of holomorphic functions and the sheaf of germs of holomorphic p-forms. We define $h^{p,q}=\dim H^q(X,\Omega^p)$, $P_m=\dim H^\circ(X,(\Omega^n)^{\otimes m})$, $r=$ the number of linearly independent closed holomorphic 1-forms, $\kappa=$ Kodaira dimension of X and $b_i=$ the i-th Betti number of X. S. Iitaka proposed the problem whether all P_m and κ are deformation-invariants ([1]).

2°. Proposition. *A simply connected complex Lie group G of $\dim_C n$ is analytically homeomorphic to $\boldsymbol{C}^n$ as a complex manifold.*

Proof. We shall prove the proposition by induction on n. It is obvious in case of $n=1$. Let the Lie group be G. If $n\geqq 2$, we can take a connected normal subgroup N. Then the canonical mapping $\pi: G\to G/N$ defines a holomorphic fiber bundle. Since both G/N and N are connected and simply connected we obtain the proposition by the induction hypothesis and Grauert's theorem.

3°. We define a complex parallelisable manifold to be solvable if the corresponding Lie group is solvable. From now on we assume X to be solvable. Note that the universal covering of X is analytically homeomorphic to $\boldsymbol{C}^n$ by the above Proposition.

Theorem 1. *Three dimensional solvable manifolds are classified into the following four classes.*

	Lie group	b_1	r	$h^{0,1}$	Structure (Albanese mapping)
(1)	abelian	6	3	3	complex torus
(2)	nilpotent	4	2	2	T^1-bundle over T^2
(3a)	solvable	2	1	1	T^2-bundle over T^1
(3b)	solvable	2	1	3	T^2-bundle over T^1