# 161. On the Boundary Value Problem for Elliptic System of Linear Differential Equations. I

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The purpose of this note is to formulate the boundary value problem for an elliptic system of linear differential equations. We present a theorem which clarifies the relation between the "cohomology groups on the boundary" and those of solutions of the equation on the ambient space (Theorem 1). We refer the reader to Sato, Kawai, and Kashiwara [7] (hereafter referred to as S-K-K) as for notions and notations used in this note. For instance, by a *system* $\mathcal{M}$ of linear differential equations defined on a real analytic manifold $M$ we mean a left $\mathcal{D}_M$-Module which is *admissible*, i.e. admits locally a representation $\mathcal{M}=\mathcal{D}_M \otimes_{\mathcal{D}_M^f} \mathcal{M}^f$ by means of a *coherent* left $\mathcal{D}_M^f$-Module $\mathcal{M}^f$, and by an *elliptic* system a system $\mathcal{M}$ for which $\mathrm{Supp}\,(\mathcal{P}\otimes_{\mathcal{D}}\mathcal{M})\cap\sqrt{-1}S^*M=\phi$. Our formulation of the problem is closely tied with the theory of microfunctions (see Example 1).

Further details of this note will appear elsewhere.

In order to state our main theorem (Theorem 1) we prepare some notations. Let $N$ be a submanifold of a real analytic manifold $M$ with codimension $d$. Let $\pi_{N/M}: {}^N\widetilde{M}^*\to M$ be the comonoidal transformation of $M$ with center $N$. Then $\pi_{N/M}^{-1}(N)$ is the conormal spherical bundle $S_N^*M$ by the definition. Let $X$ and $Y$ be a complex neighborhood of $M$ and $N$ respectively. Note that the conormal bundle $T_N^*X$ is a direct sum $T_N^*M\oplus\sqrt{-1}T^*M$ by the complex structure of $X$. We denote by $p$ the canonical projection from $S_N^*X-S_Y^*X$ to $S_N^*Y=\sqrt{-1}S^*N$ and by $q$ the canonical projection from $S_N^*X-\sqrt{-1}S^*M$ to $S_N^*M$. $\omega_{N/M}$ denotes the locally constant sheaf $\mathcal{H}_N^d(\boldsymbol{C}_M)$.

**Theorem 1.** *Let $\mathcal{M}$ be an elliptic system of linear differential equations on $M$. Assume that $N$ is non-characteristic with respect to $\mathcal{M}$. Setting $\mathcal{S}=\boldsymbol{R}\,\mathcal{H}om_{\mathcal{D}_M}(\mathcal{M},\mathcal{B}_M)=\boldsymbol{R}\,\mathcal{H}om_{\mathcal{D}_M}(\mathcal{M},\mathcal{A}_M)$, we have the following canonical isomorphism*:

$$(1)\qquad \boldsymbol{R}\Gamma_{S_N^*M}(\pi_{N/M}^{-1}\mathcal{S})\otimes\omega_{N/M}[d]\cong\boldsymbol{R}q_*\boldsymbol{R}\,\mathcal{H}om_{p^{-1}\mathcal{P}_N}\Big(\mathcal{P}_{\dot{Y}\to X}\underset{\mathcal{D}_X}{\otimes}\mathcal{M}|_{S_N^*X},\ p^{-1}C_N\Big)$$

Important applications, which will bring the deep meaning of the theorem tangible, are given in our subsequent papers. It may seem, however, that our formulation above be too general and abstract, so