160. *On Green's Functions of Elliptic and Parabolic Boundary Value Problems*

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1. Introduction. Let $A(x, D)$ be an elliptic operator of order m defined in a domain Ω of R^n, and $B_j(x, D)$, $j=1, \cdots, m/2$, be operators of order $m_j<m$ defined on $\partial\Omega$. We assume

(i) the system $(A(x, D), \{B_j(x, D)\}_{j=1}^{m/2}, \Omega)$ as well as its adjoint system $(A'(x, D), \{B'_j(x, D)\}_{j=1}^{m/2}, \Omega)$ formally constructed are both regular systems in the sense of S. Amon [1];

(ii) there is an angle $\theta_0 \in (0, \pi/2)$ such that $(e^{i\theta}D_t^m - A(x, D_x)$, $\{B_j(x, D_x)\}_{j=1}^{m/2}$, $\Omega\times(-\infty<t<\infty))$ is an elliptic boundary value problem satisfying the coerciveness condition for any $\theta \in [\theta_0, 2\pi-\theta_0]$ (cf. S. Agmon [1]).

Let A be the operator defined by

$$D(A)=\{u \in H_m(\Omega): B_j(x, D)u=0 \text{ on } \partial\Omega, j=1, \cdots, m/2\}$$

and $(Au)(x)=A(x, D)u(x)$ for $u \in D(A)$. It is known that the operator defined analogously by the adjoint system $(A'(x, D), \{B'_j(x, D)\}, \Omega)$ coincides with the adjoint of A (F.E. Browder [5], [6]).

In this paper we describe a method of establishing global estimates for the Green's function of the resolvent of A as well as the semigroup $\exp(-tA)$ generated by $-A$. Under the present assumptions the resolvent $(A-\lambda)^{-1}$ exists for λ in the set defined by $\Lambda=\{\lambda: \theta_0 \leqq \arg\lambda \leqq 2\pi-\theta_0, |\lambda|>C_0\}$ for some $C_0>0$ ([1]) and $-A$ generates a semigroup which is analytic in the sector $\Sigma=\{t: |\arg t|<\pi/2-\theta_0\}$.

Theorem 1. *Let $K_\lambda(x, y)$ be the kernel of $(A-\lambda)^{-1}$. Then there exist constants C and $\delta>0$ such that*

(a) $|K_\lambda(x, y)| \leqq Ce^{-\delta|\lambda|^{1/m}|x-y|}|\lambda|^{n/m-1}$ *if* $m>n$,

(b) $|K_\lambda(x, y)| \leqq Ce^{-\delta|\lambda|^{1/m}|x-y|}|x-y|^{m-n}$ *if* $m<n$,

(c) $|K_\lambda(x, y)| \leqq Ce^{-\delta|\lambda|^{1/m}|x-y|}\{1+\log^+(|\lambda|^{-1/m}|x-y|^{-1})\}$ *if* $m=n$

for $x, y \in \Omega$ and $\lambda \in \Lambda$.

Theorem 2. *Let $G(x, y, t)$ be the kernel of* $\exp(-tA)$. *Then there exist positive constants C and c such that*

$$|G(x, y, t)| \leqq C|t|^{-n/m}\exp(-c|x-y|^{m/(m-1)}/|t|^{1/(m-1)})e^{C|t|}$$

for $x, y \in \Omega$ and $t \in \Sigma$.

Remark 1. The boundedness of Ω is required in the assumption (i); however, it is not essential. The same results remain valid if Ω is an unbounded domain uniformly regular of class C^m and locally regular