9. *The Second Dual Space for the Space* N^{+}

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1. Introduction. Let D be the unit disk $\{|z|<1\}$. A holomorphic function $f(z)$ in D is said to belong to the *class* N of functions of bounded characteristic if

$$T(r,f)=\frac{1}{2\pi}\int_0^{2\pi}\log^+|f(re^{i\theta})|\,d\theta=O(1)\text{ as } r\to 1. \tag{1.1}$$

A function $f(z)\in N$ is said to belong to the *class* N^+ [2, p. 25] if

$$\lim_{r\to 1}\int_0^{2\pi}\log^+|f(re^{i\theta})|\,d\theta=\int_0^{2\pi}\log^+|f(e^{i\theta})|\,d\theta. \tag{1.2}$$

We showed in [7] that the class N^+ becomes an F-space in the sense of Banach [1, p. 51] with the distance function

$$\rho(f,g)=\frac{1}{2\pi}\int_0^{2\pi}\log(1+|f(e^{i\theta})-g(e^{i\theta})|)d\theta \tag{1.3}$$

The space N^+ with this metric (1.3) is not locally convex and not locally bounded [7, corollary to Theorem 2]. But N^+ has sufficiently many continuous linear functionals to form a dual system $\langle (N^+)^*, N^+\rangle$ in the sense of Dieudonné and Mackey [5, p. 88].

Duren, Romberg, and Shields [3] studied the dual space $(H^p)^*$ of $H^p, 0<p<1$, and defined the containing Banach space $B^p\subset(H^p)^{**}$. Treating the corresponding problems for N^+, instead of H^p, we defined the containing Fréchet space F^+ for the class N^+ [8]. We will show in this note that F^+ is nothing but the second dual $(N^+)^{**}$ of N^+, and will obtain some results on its properties.

2. The space $(N^+)^{**}$. We denote by S the collection of complex sequences $\{b_n\}$ such that

$$\overline{\lim_{n\to\infty}}\{(1/\sqrt{n})\log^+|b_n|\}<0. \tag{2.1}$$

(2.1) means: there are constants $K=K(\{b_n\})$, $c=c(\{b_n\})>0$ such that

$$|b_n|\leqq K\exp[-c\sqrt{n}]. \tag{2.2}$$

In [7, Theorem 3] we showed:

Let ϕ be a continuous linear functional on N^+. Then there is a unique holomorphic function $g(z)=\sum b_n z^n$, continuous on $\bar{D}$, such that for any $f(z)=\sum a_n z^n\in N^+$

$$\begin{aligned}\phi(f)&=\lim_{r\to 1}\frac{1}{2\pi}\int_0^{2\pi}f(re^{i\theta})g(e^{-i\theta})d\theta\\&=\sum_{n=0}^{\infty}a_n b_n\text{ (absolutely convergent)}\end{aligned} \tag{2.3}$$