5. On Continuation of Regular Solutions of Partial Differential Equations with Constant Coefficients

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This is a short communication of the results of my forthcoming paper [4]. Let $\mathcal{A}$ (resp. $\mathcal{B}$) be the sheaf of real analytic functions (resp. that of hyperfunctions). Let $p(D)$ be a partial differential equation with constant coefficients, and let $\mathcal{A}_p$ (resp. $\mathcal{B}_p$) be the sheaf of real analytic solutions (resp. that of hyperfunction solutions) of $p(D)u=0$. In our earlier work [2], we have given the condition for the operator p in order that $\mathcal{A}_p(U\backslash K)/\mathcal{A}_p(U)=0$, where K is a compact convex subset of $\boldsymbol{R}^n$ and U is one of its open convex neighborhoods. Now let K be the intersection of a compact convex set with the open half space $\{x_n<0\}$ in $\boldsymbol{R}^n$, and let U be one of its open convex neighborhoods. Here, we employ the coordinates $(x_1, \cdots, x_n)=(x', x_n)$ for $\boldsymbol{R}^n$. Concerning the possibility of extension of the solutions of $p(D)u=0$ in $U\backslash K$ to the whole U, we have the following results.

Theorem 1. *$\mathcal{B}_p(U\backslash K)/\mathcal{B}_p(U)=0$ if and only if*

$$H_L(\zeta)\leq\varepsilon|\zeta|+H_{L\backslash K}(\zeta)+C_\varepsilon, \qquad \text{for } \zeta\in N(p), \ (\forall\varepsilon>0, \ \exists C_\varepsilon>0).$$

Here L is the closure of K in $\boldsymbol{R}^n$, $H_L(\zeta)=\sup_{x\in L}\operatorname{Re}\langle x, \sqrt{-1}\zeta\rangle$ is its supporting function and similarly for $H_{L\backslash K}(\zeta)$; $N(p)$ is the characteristic variety $\{\zeta\in C^n; p(\zeta)=0\}$ of p.

We can easily prove that the restriction map $\mathcal{B}_p(U)\to\mathcal{B}_p(U\backslash K)$ is injective. Therefore the factor space $\mathcal{B}_p(U\backslash K)/\mathcal{B}_p(U)$ is well defined.

Corollary 2. *If $\mathcal{B}_p(U\backslash K)/\mathcal{B}_p(U)=0$, then p is hyperbolic with respect to the direction $(0, \cdots, 0, 1)$. Conversely, let p be hyperbolic to that direction. Then, for each K which is the part in $\{x_n<0\}$ of a cone with x_n-axis as its axis and with a sufficiently mild vertical angle, we have $\mathcal{B}_p(U\backslash K)/\mathcal{B}_p(U)=0$.*

Here we mean hyperbolicity in the sense of hyperfunctions (see [5], Definition 6.1.1). These results are obtained by cohomological arguements for $\mathcal{B}_p$ and by applying the fundamental principle for $\mathcal{B}_p$ established in [2], II. Note that the possibility of extension of hyperfunction solutions really depends on the shape of K.

As for real analytic solutions we get the following result immediately form Corollary 2, when we take into account the result on

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