4. *A Remark on Integral Equation in a Banach Space*

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1. Introduction and main theorem.

The main object of this paper is to extend the result of G. Webb [1] on the solution of the integral equation associated with some nonlinear equation of evolution in a Banach space to the time dependent case.

Let E be a Banach space with norm $\| \quad \|$.

Let $A(t)$ $(0 \leqq t \leqq T)$ be a linear accretive operator which satisfies the conditions of T. Kato [2], H. Tanabe [3] or T. Kato and H. Tanabe [4], and $B(t)$ be a nonlinear, accretive, everywhere defined operator such that $(t, u) \to B(t)u$ is a strongly continuous mapping from $[0, T] \times E$ to E which maps bounded sets to bounded sets. It is known that there exists an evolution operator $U(t, \tau)$ $0 \leqq \tau \leqq t \leqq T$ with norm $\leqq 1$ to the linear equation $du(t)/dt + A(t)u(t) = 0$, and that $A(t)$ is m-accretive for $t \in [0, T]$.

Then we can state our main theorem.

Theorem. *Under our assumption, for any $x \in E$, $\tau \in [0, T[$, there exists a unique solution $u(t, \tau; x)$ to the integral equation*

$$(E) \qquad u(t, \tau; x) = U(t, \tau)x - \int_{\tau}^{t} U(t, s)B(s)u(s, \tau; x)ds$$

on $[\tau, T]$. If we define $W(t, \tau)x = u(t, \tau; x)$, then $W(t, \tau)$ has the following evolution properties and an inequality,

(1) $W(t, \tau) = W(t, t')W(t', \tau)$, $W(t, t) = I$ *for* $0 \leqq \tau \leqq t' \leqq t \leqq T$

(2) $W(t, \tau)x$ *is strongly continuous in* $0 \leqq \tau \leqq t \leqq T$

(3) $\| W(t, \tau)x - W(t, \tau)y \| \leqq \| x - y \|$

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2. Proof of the theorem.

The main idea of the proof is due to G. Webb [1].

Proposition 1. *For any $x \in E$, $\tau \in [0, T[$, there exists $T_0 (\tau < T_0 \leqq T)$ and a continuous function $u(t, \tau; x): [\tau, T_0] \to E$ such that $u(t, \tau; x)$ is a solution of (E) on $[\tau, T_0]$.*

Proof. Let $x \in E$, $\tau \in [0, T[$ be fixed. In view of the continuity of $B(t)x$, for any $\varepsilon > 0$ there exists a positive number δ depending on x, τ, ε, such that for any $v \in V = \{v : \| x - v \| < \delta\}$ and any $t, |t - \tau| < \delta$ the inequality $\| B(t)v - B(\tau)x \| \leqq \varepsilon$ hold. Take $M = \| B(\tau)x \| + \varepsilon$ then $\| B(t)v \| \leqq M$ for any $v \in V$ and $t, |t - \tau| < \delta$. Under the assumptions of [2] or [3] we