2. On the Relative Pseudo-Rigidity

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(Comm. by Kunihiko KODAIRA, M. J. A., Jan. 12, 1973)

In this paper we establish a generalization of the results in [1].

In what follows, by a *pair* (W, S) we mean the pair of a complex manifold W and a compact submanifold S of W. By a *deformation* of a pair (W, S) we mean the quintuple $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ of connected complex manifolds $\mathcal{W}, B$, a closed submanifold $\mathcal{S}$ of $\mathcal{W}$, a point o of B and a smooth holomorphic map π of $\mathcal{W}$ onto B such that $\pi^{-1}(o)=W$, $\pi^{-1}(o)\cap\mathcal{S}=S$ and the restriction of π to $\mathcal{S}$ is a proper smooth holomorphic map.

For convenience sake we list here some notations whose meanings are the same wherever they occur. Let $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ be a deformation of a pair (W, S).

$m=\dim B$

$(t_1, \cdots, t_m)=$a local coordinate of B with center o

$B(\varepsilon)=\{(t_1, \cdots, t_m)\in B\,;\,|t_i|<\varepsilon,\ i=1, \cdots, m\}$

$\mathcal{W}(\varepsilon)=\pi^{-1}(B(\varepsilon))$

$\mathcal{W}|_U=\pi^{-1}(U)$, $U\subset B$

$\Xi=$the sheaf over W of germs of holomorphic vector fields which are tangential to S at each point of S.

$\tilde{\Xi}=$the sheaf over $\mathcal{W}$ of germs of holomorphic vector fields along fibres which are tangential to $\mathcal{S}$ at each point of $\mathcal{S}$.

We say that a deformation $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ of a pair (W, S) is *relatively trivial* if there exists a biholomorphic map g of $\mathcal{W}$ onto $W\times B$ which induces a biholomorphic map of $\mathcal{S}$ onto $S\times B$ such that $g|W$ is the identity map and $pr_B\circ g=\pi$ where pr_B is the canonical projection of $W\times B$ onto B.

Definition 1. A deformation $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ of a pair (W, S) is said to be *relatively pseudo-trivial at o* if, for any relative compact subset N of W, there exist a positive number ε and a submanifold $\mathcal{N}$ of $\mathcal{W}(\varepsilon)$ such that $(\mathcal{N}, \mathcal{N}\cap\mathcal{S}, B(\varepsilon), o, \pi|\mathcal{N})$ is a relative trivial deformation of the pair $(N, N\cap S)$.

Definition 2. A pair (W, S) is said to be *relatively pseudo-rigid* if any deformation of (W, S) is relatively pseudo-trivial at o.

Lemma. *Let* $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ *be a deformation of a pair* (W, S). *If the stalk* $(R^1\pi_*\tilde{\Xi})_o=0$, *then* $(\mathcal{W}, \mathcal{S}, B, o, \pi)$ *is relatively pseudo-trivial at* o.