30. *A Note on a Problem of Matlis*

By Kunio YAMAGATA
Tokyo University of Education

(Comm. by Kenjiro SHODA, M. J. A., Feb. 12, 1973)

Following Faith and Walker [2] a module is said to be completely decomposable if it is a direct sum of indecomposable injective submodules. And a right ideal I of a ring R is called irreducible if $I \neq R$ and $I = I_1 \cap I_2$ implies $I = I_1$ or $I = I_2$, for all right ideals I_1 and I_2 of R.

It is an open problem whether every direct summand of a completely decomposable module is also completely decomposable, and E. Matlis [5] proved that we have an affirmative answer for modules over a right Noetherian ring. Recently in [6] we have proved that if a ring is non-singular and satisfying the ascending chain condition for essential right ideals its answer is also in the affirmative. Further it is known by us that the non-singular condition of them can be removed. Thus, in this note, using a result of Harada and Sai [3], we shall prove it as a corollary to the theorem which is a special case, concerning the completely decomposable modules, of the Krull—Remak—Schmidt—Azumaya's theorem. Namely,

Theorem 1. *The following conditions are equivalent.*

(I) *A ring R satisfies the ascending chain condition for irreducible right ideals.*

(II) *A ring R satisfies the ascending chain condition for essential, irreducible right ideals.*

(III) *If a completely decomposable module M_R has two direct sum decompositions in which each component is indecomposable, injective submodule;*

$$M = \sum_{i \in I} \oplus M_i = \sum_{j \in J} \oplus N_j,$$

then for any subset $I' \subset I$ (resp. $J' \subset J$) *there exists a one-to-one mapping φ of I' into J* (resp. J' *into* I) *such that $M_i \cong N_{\varphi(i)}$ for all $i \in I'$* (resp. $N_j \cong M_{\varphi(j)}$ *for all* $j \in J'$) *and*

$$M = \sum_{i \in I'} \oplus N_{\varphi(i)} \oplus \sum_{i \in I - I'} \oplus M_i$$

$$\left(\text{resp. } M = \sum_{j \in J'} \oplus N_j \oplus \sum_{i \in I - \varphi(J')} \oplus M_i\right).$$

Corollary. *If a ring satisfies the equivalent condition in Theorem 1, then every direct summand of a completely decomposable module is also completely decomposable.*

In case a ring R is right Noetherian the theorem is a part of [3;