28. Generalized Prime Elements in a Compactly Generated l-Semigroup. I

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In [6] by introducing f-systems authors have defined f-prime ideals in rings as a generalization of prime ideals [2] and s-prime ideals [8], and generalized under certain assumptions usual decomposition theorems of ideals and the concept of relatedness in general rings [2], [3], [7], [8]. The aim of the present note is to present similar results for "elements" of an l-semigroup with some restricted compact generator system. The results obtained here are applicable for general rings and some kind of algebraic systems.

1. Mapping φ, φ-Prime Elements.

Let L be a cm-lattice [1] with the following four conditions:

(1) L has the greatest element e.

(2) L has the least element 0.

(3) Both ae and ea are less than a, i.e. $ae \leq a$ and $ea \leq a$.

(4) L has a compact generator system [4].

It is then easy to see that $a0=0a=0$, $ab \leq a$ and $ab \leq b$ for any a, b in L. If in particular e is unity quantity, the condition (3) is superfluous. From now on Σ will denote a compact generator system of L, $\Sigma(a)$ the set of the compact elements (elements in Σ) which are less than a, and $\Sigma'(a)$ the complement of $\Sigma(a)$ in Σ. Throughout this note we suppose that

(*) *if* $u \in \Sigma(a \cup b)$, *there exists an element* x *of* $\Sigma(a)$ *such that* $\Sigma(x \cup b) \ni u$, *where* a, b *are in* L.

Let R be an associative or nonassociative ring (or more generally a ringoid [1]), and let L_R, Σ_R and Σ_R^* be the sets of all (two-sided) ideals of R, of all principal ideals of R and of all finitely generated ideals of R, respectively. Then it can be shown that L_R is a cm-lattice with (1), (2), (3) and (4). It is easy to see that Σ_R is a compact generator system with the condition (*). Similarly for Σ_R^*. Let G be an arbitrary group, and let L_G, Σ_G and Σ_G^* be the sets of all normal subgroups of G, of all normal subgroups with single generators and of all finitely generated normal subgroups of G, respectively. Then it can be shown that L_G is a cm-lattice under inclusion relation and commutator-product. It is then easily verified that the conditions (1), (2), (3) and (4)

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