27. On the Euler-Characteristic and the Signature of G-Manifolds*)

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§0. Let W be a closed Riemann surface. A conformal self map of W will be called an automorphism. If G is a finite group of automorphisms of W, then the orbit space W/G is naturally a Riemann surface. In [1], [2] R. D. M. Accola proved certain formulas which relate the genera of W, W/G and W/H where H ranges over certain subgroups of G. He proved them using the Riemann-Hurwitz formula for the coverings $W \to W/G$ and $W \to W/H$.

The purpose of this note is to extend his results. In §1 we shall prove formulas in the case of the Euler-characteristic of compact Hausdorff spaces on which a finite group G acts as the group of homeomorphisms. In §2 we shall prove a formula in the case of the signature of closed connected oriented generalized $4k$-dimensional manifolds over the field of real numbers on which a finite group G acts effectively and orientation preservingly as the group of homeomorphisms.

§1. Throughout this section let X be a compact Hausdorff space on which a finite group G acts and let the cohomology group $H^*(X)$ of X be the Céch cohomology group with real coefficients. Moreover let the groups $H^n(X)$ be finite dimensional, and zero for $n > i$ (i is some integer). Since $H^*(X)$ is naturally a G-module, we have the submodule $H^*(X)^G$ consisting of all invariant elements of $H^*(X)$. Let X/G denote the orbit space and $p: X \to X/G$ the projection. Then the following lemma is known [3].

Lemma 1. *The homomorphism $p^*: H^n(X/G) \to H^n(X)$ is the monomorphism and its image is $H^n(X)^G$.*

Define a homomorphism $\varphi: H^n(X) \to H^n(X)$ by

$$\varphi(\alpha) = \frac{1}{|G|} \sum_{g \in G} g^*(\alpha) \qquad (\alpha \in H^n(X)).$$

Then it is easily seen that α is in $H^n(X)^G$ if and only if $\varphi(\alpha) = \alpha$. Therefore it holds that

$$\dim H^n(X)^G = \operatorname{trace} \varphi$$
$$= \frac{1}{|G|} \sum_{g \in G} \operatorname{trace}\,(g^*: H^n(X) \to H^n(X)).$$

*) Dedicated to Professor Shigeo Sasaki on his 60th birthday.