24. On a Generalization of Adasch's Theorem

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N. Adasch [1] generalized Köthe's theorem [4] on the equicontinuous set of linear continuous mappings from (F)-space into (LF)-space. In this paper, we shall go a step further.

In the first part, we introduce the concept of the countably boundedness which generalizes the theorem ([1](10)b)⇒c)). Next, we generalize the theorem ([1](12)a)⇒b)) in the second part and the theorems ([1](10)c)⇒b)), ([1](12)b)⇒a)) in the third part.

Throughout this paper, terminology and notation are the same as in [3], if nothing otherwise is mentioned.

1. **Definition 1.** Let E be a locally convex separative topological linear space and A a subset of it. We say that A is countably bounded if, for any sequence $\{x_n\}$ of elements of A, there exists an absolutely convex bounded set B of E such that $\{x_n\}\subset E_B$. When E is countably bounded, E is said to be countably bounded space.

We have easily next seven propositions.

Proposition 1. *Any bounded subset of a locally convex separative topological linear space is countably bounded.*

Proposition 2. *Any finite union or sum of countably bounded subsets and any subset of a countably bounded set is countably bounded. Especially, any subspace of a countably bounded space is a countably bounded space for the induced topology.*

Proposition 3. *Any finite product of countably bounded spaces is countably bounded. A countably bounded space E is countably bounded for the topology such that E has the same dual E'.*

Proposition 4. *If a locally convex separative topological linear space E has the first countability property of Mackey* [5], [*Proposition* 12 *of this paper*], *then E is countably bounded. Especially, every metrizable locally convex topological linear space is countably bounded.*

Proposition 5. *Let E be a locally convex separative topological linear space and B an absolutely convex bounded set of E. Then E_B is a countably bounded space for the topology of E_B, and the induced topology.*

Proposition 6. *Let E be the locally convex separative topological linear space which is the union of a sequence of linear subspaces $\{E_n\}$. Then the following assertions are equivalent:*