23. *On Symmetric Spaces*

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1. Introduction. A. V. Arhangel'skii [1] has introduced the notion of symmetric spaces and given many interesting results in the theory for metrizability and so on. In this note, we shall discuss some properties concerning symmetric spaces: symmetrizability of subspaces, local properties, products of symmetric spaces and images of symmetric spaces under suitable maps.

We assume all spaces are Hausdorff and all maps are continuous and onto. We denote by 2^X the collection of all subsets of X, and abbreviate by $\{x_i\}$ a sequence $\{x_i; i=1, 2, \cdots\}$.

2. Preliminaries. We begin by recording definitions of symmetric spaces and related spaces.

Definition 2.1. A space X is *symmetric* if there is a real valued, non-negative function d defined on $X \times X$ satisfying the conditions: (1) $d(x, y)=0$ whenever $x=y$, (2) $d(x, y)=d(y, x)$, (3) $A \subset X$ is closed in X whenever $d(x, A)>0$ for any $x \in X-A$.

If we replace the condition (3) by the following: For $A \subset X$, $x \in \overline{A}$ whenever $d(x, A)=0$, then such a space is called *semi-metric* [9].

A space X is *sequential* if $A \subset X$ is closed whenever $A \cap C$ is closed for every compact metric subset C of X [6].

A space X is *θ-refinable* if for each open covering $\mathfrak{U}$ of X, there is a sequence $\{\mathfrak{U}_i\}$ of open refinements of $\mathfrak{U}$ such that, for $x \in X$, there is an open covering $\mathfrak{U}_i$ which is finite at x [15].

As is well known, we have the relations between such spaces and other spaces as follows:

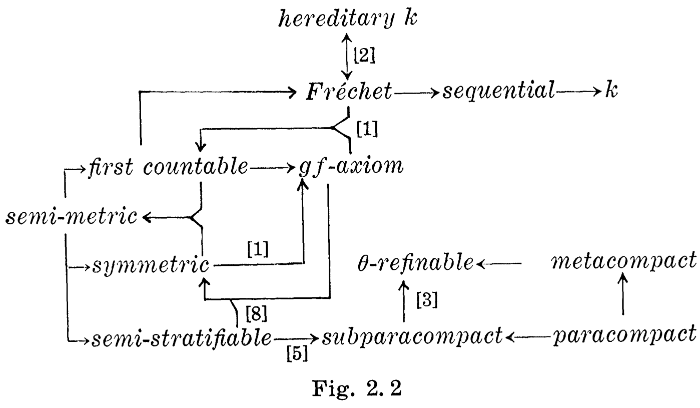

Fig. 2. 2