21. *On the Boundedness of a Class of Operator-valued Pseudo-differential Operators in L^p Space*

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Introduction. In this paper we present a class of pseudo-differential operators which are continuous in $L^p(\mathbf{R}^n)$, $1<p<\infty$. They will play an important role in studying the complex interpolation spaces of Sobolev spaces (see [3]).

Our main tools are the operator-valued version of Calderón-Vaillancourt's L^2-boundedness theorem ([2]), the Marcinkiewicz interpolation theorem, and the real-variable technique of Calderón and Zygmund which gives the weak-type estimate.

Notations. $\mathcal{L}(X, Y)$—the space of bounded linear operators from a Banach space X to a Banach space Y.

$L^p(E, d\mu; X)$—the space of X-valued L^p functions on a measure space $(E, d\mu)$

$$L^p(\mathbf{R}^n; X)=L^p(\mathbf{R}^n, dx; X), \qquad L^p(E, d\mu)=L^p(E, d\mu; \mathbf{C}).$$

$$x=(x_1, \cdots, x_n) \in \mathbf{R}^n, \qquad \alpha=(\alpha_1, \cdots, \alpha_n), \alpha_j \text{ are integers},$$

$$x^\alpha=x_1^{\alpha_1}\cdots x_n^{\alpha_n}, \qquad |\alpha|=\alpha_1+\cdots+\alpha_n,$$

$$|x|^2=x_1^2+\cdots+x_n^2, \quad D^\alpha=D_1^{\alpha_1}\cdots D_n^{\alpha_n}, \quad D_j=\partial/\partial x_j.$$

$\mathcal{S}(\mathbf{R}^n; X)$—the space of X-valued rapidly decreasing C^∞ functions.

$m(S)$—measure of the set $S\subset \mathbf{R}^n$. $a_n=m\{x \mid |x|\leqq 1\}$.

Definition. Let X, Y be two Banach spaces. Then an $\mathcal{L}(X, Y)$-valued infinitely differentiable function $p(x, \xi, y)$ of $(x, \xi, y) \in \mathbf{R}^n\times\mathbf{R}^n\times\mathbf{R}^n$ belongs to $S^\mu_{\rho,\delta,\varepsilon}(\mathbf{R}^{3n}, X; Y)$ if

$$\|D_x^\alpha D_\xi^\beta p(x, \xi, y)\|_{\mathcal{L}(X,Y)}\leqq C(1+|\xi|)^{\mu+\delta|\alpha|-\rho|\beta|}, \tag{1}$$

$$\|D_y^\gamma D_\xi^\beta p(x, \xi, y)\|_{\mathcal{L}(X,Y)}\leqq C(1+|\xi|)^{\mu+\varepsilon|\gamma|-\rho|\beta|}, \tag{2}$$

for any multi-index α, β, γ, where $0\leqq \rho, \delta, \varepsilon \leqq 1$.

For any p of this kind with $\varepsilon<1$ and for any $f \in \mathcal{S}(\mathbf{R}^n; X)$ the integral

$$\begin{aligned} Tf(x)&=\frac{1}{(2\pi)^n}\int e^{ix\xi}d\xi\int p(x, \xi, y)f(y)e^{-i\xi y}dy \\ &=\frac{1}{(2\pi)^n}\int (1+|\xi|^2)^{-m}(1-\Delta_y)^m\{p(x, \xi, y)f(y)\}e^{i\xi(x-y)}d\xi dy \end{aligned}$$

is well defined and Tf belongs to $\mathcal{S}(\mathbf{R}^n; Y)$, where m is a positive integer such that $2m(1-\varepsilon)>\mu+n$, and Δ_y the Laplacian operator.

Theorem 1. *Let X, Y be two Hilbert spaces,*