20. On the Existence of Hyperfunction Solutions of Linear Differential Equations of the First Order with Degenerate Real Principal Symbols

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0. Introduction. Let P be a first order linear partial differential operator of the form $P=\sum_{i,j=1}^{n} a_j^i x_j \partial/\partial x_i$, where $(a_j^i) \in GL(n, \boldsymbol{R})$. In this note we study the problem of existence of hyperfunction solutions of the equation $Pu=f$ mainly in the case where $n=2$, and show that the local solvability is valid in some cases. It is easy to see that P is locally solvable except in the neighborhood of the origin. On the other hand, the principal symbol of P vanishes at the origin. Hence our main interest is in the local solvability of the equation at the origin. By the flabbiness of the sheaf $\mathcal{B}$ of hyperfunctions we can deduce the local solvability from the global solvability, while we will show later that $P\mathcal{B}(\boldsymbol{R}^n)=\mathcal{B}(\boldsymbol{R}^n)$ in some cases.

H. Suzuki [4] has studied the local solvability of linear partial differential equations of the first order in two independent variables where the principal symbols do not vanish. He uses characteristic curves in the complex domain. Recently Suzuki [5] generalized Lemma 2 in Suzuki [4] as follows:

Theorem 0.1. *Let V be a domain of holomorphy and $P=\partial/\partial x_1$. Then the following are necessary and sufficient conditions for $P\mathcal{O}(V)=\mathcal{O}(V)$;*

(a) *For every $x \in V$, L_x is simply connected, where L_x is the connected component of the set $\{x' \in V; x'_i = x_i \text{ for } i=2, \cdots, n\}$ which contains the point x.*

(b) *The topology of V/P is Hausdorff, where V/P is the quotient space of V by the equivalence relation "$L_x = L_{x'}$" $(x, x' \in V)$.*

(c) *V/P is a domain of holomorphy over $\boldsymbol{C}^{n-1}$.*

We shall mainly use the same method as Suzuki [4] and rely on Theorem 0.1. But in one case we have to employ a different method (Case 1). The reason why we have to use two different methods may be explained by the theory of the sheaf $\mathcal{C}$. The author expresses his gratitude to Mr. Kashiwara for this suggestion. Lastly we remark that in the case of ordinary differential operators the problem has been completely solved by Sato [3] and Komatsu [1].