19. *On the Theorem of Cauchy-Kowalevsky for First Order Linear Differential Equations with Degenerate Principal Symbols*

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Let

(1) $$P=\sum_{i=1}^{n} a_i(x)\frac{\partial}{\partial x_i}+b(x)$$

be a first order linear differential operator with analytic coefficients defined at the origin of C^n. In this note, we discuss the following problem: Consider the differential equation

(2) $$Pu=f.$$

f and u being analytic functions at the origin, what condition should f satisfy for the existence of a local solution u of the equation (2) and how many solutions exist when f satisfies the condition? That is, our problem is to clarify the kernel and cokernel of the operator P. When $n=1$, Komatsu [2] and Malgrange [3] have a deep result for the index of the operator P, which is not necessarily of the first order.

Let $\mathcal{O}$ be the stalk at the origin of the sheaf of holomorphic functions over C^n. Let $\mathfrak{A}$ and $\mathfrak{B}$ be the ideals of $\mathcal{O}$ generated by $a_1(x), \cdots, a_n(x)$ and $a_1(x), \cdots, a_n(x), b(x)$ respectively. In the case when $\mathfrak{A}$ is equal to $\mathcal{O}$, the answer to this problem is well-known as the theorem of Cauchy-Kowalevsky. In this note, therefore, we assume that $\mathfrak{A}$ is a proper ideal of $\mathcal{O}$. Such equations are used by Hadamard [1] to construct the elementary solution of a second order linear partial differential equation and by Sato-Kawai-Kashiwara [4] to determine the structure of pseudo-differential equations. We want to have general theory about the equation of such type. First we give the following conditions to formulate a theorem. We discuss examples which do not satisfy these conditions later.

(A) *$\mathfrak{A}$ is a proper and simple ideal of $\mathcal{O}$.*

Let $M=(\partial(a_1, \cdots, a_n)/\partial(x_1, \cdots, x_n))(0)$ be the Jacobian matrix of $a_1, \cdots, a_n$ at the origin. Let $M^*=J_1\oplus\cdots\oplus J_m\oplus J'_1\oplus\cdots\oplus J'_{m'}$ be the Jordan canonical matrix of M, where $J_i(1\leqslant i\leqslant m)$ and $J'_j(1\leqslant j\leqslant m')$ are the matrices of the Jordan blocks of sizes N_i and N'_j with eigenvalues $\lambda_i\neq 0$ and $\lambda'_j=0$ respectively.

(B) i) $N'_j=1$ $(1\leqslant j\leqslant m')$.