55. *A Remark on the Normal Expectations. II*

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1. In the previous note [3], the concept of generalized channels is introduced. In the note [2], it is proved that, for a von Neumann algebra and a von Neumann subalgebra of it, the conjugate mapping of a generalized channel with a certain property is a normal expectation.

In this note, we shall show that a generalized channel is considered a normal expectation.

2. Consider a von Neumann algebra $\mathcal{A}$, denote the conjugate space of $\mathcal{A}$ as $\mathcal{A}^*$ and the subconjugate space of all ultra-weakly continuous linear functionals on $\mathcal{A}$ as $\mathcal{A}_*$, following after the definition of Dixmier [4].

Definition (cf. [3]). Let $\mathcal{A}$ and $\mathcal{B}$ be two von Neumann algebras, then a positive linear mapping π of $\mathcal{A}_*$ into $\mathcal{B}_*$ is called a *generalized channel* if π maps a normal state to a normal state.

The following proposition is obtained in [3]:

Proposition 1. *A positive linear mapping π of $\mathcal{A}_*$ into $\mathcal{B}_*$ is a generalized channel if and only if the conjugate mapping π^* is a positive normal linear mapping of $\mathcal{B}$ into $\mathcal{A}$ preserving the identity.*

In the sequel, according to this proposition, a normal positive linear mapping of a von Neumann algebra into a von Neumann algebra preserving the identity will be called also a generalized channel.

Let $\mathcal{A}$ be a von Neumann algebra and $\mathcal{B}$ a von Neumann subalgebra of $\mathcal{A}$, then a positive linear mapping e of $\mathcal{A}$ onto $\mathcal{B}$ is called an *expectation* of $\mathcal{A}$ onto $\mathcal{B}$ if e satisfies the following conditions:

(i) $1^e=1$, and

(ii) $(BAC)^e=BA^eC$ for all $A\in\mathcal{A}$ and $B, C\in\mathcal{B}$, cf. [5].

The following proposition is proved in [2]:

Proposition 2. *Let $\mathcal{A}$ be a von Neumann algebra and $\mathcal{B}$ a von Neumann subalgebra of $\mathcal{A}$, then a mapping π of $\mathcal{B}_*$ to $\mathcal{A}_*$ is a generalized channel with*

(1) $$\pi L_B=L_B\pi \qquad \text{for any } B\in\mathcal{B}$$

if and only if the conjugate mapping e of $\mathcal{A}$ onto $\mathcal{B}$ is a normal expectation, where a mapping L_A on $\mathcal{A}^*$ is defined for $A\in\mathcal{A}$ by

(2) $$L_Af(X)=f(AX) \qquad \text{for all } f\in\mathcal{A}^* \text{ and } X\in\mathcal{A}.$$

Let $\mathcal{A}\otimes\mathcal{B}$ be the tensor product of von Neumann algebras $\mathcal{A}$ and