54. A Remark on the Flow near a Compact Invariant Set

By Jirō EGAWA
Kobe University

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Introduction. The qualitative behavior of the flow near a compact invariant set was studied by several authors. For planer dynamical systems, I. Bendixson gave a solution to the problem of possible qualitative behavior of the flow near a rest point. For a dynamical system on a locally compact space, T. Ura and I. Kimura gave a description of the flow near a compact invariant set ([7]). Further discussions of the flow near a compact invariant set and/or a compact minimal set were done by T. Ura, T. Saito, N. P. Bhatia and D. Desbrow, always assuming the phase space is locally compact ([1]–[3], [5], [6], [8]). In this paper we shall give an example showing that the description as in [7] is not valid if we replace the assumption "locally compact" by the assumption "complete metric" on the phase space. This example also shows that the same is true for the discussions mentioned above.

1. Notations and basic theorems.

Let π be a dynamical system on a topological space X. $C(x), C^+(x)$ and $C^-(x)$ denote the orbit, the positive semiorbit and the negative semiorbit, respectively, through a point $x \in X$. The positive (negative) limit set of $x \in X$ is denoted by $L^+(x)$ $(L^-(x))$. $\phi \neq M \subset X$ is called an invariant set if $C(x) \subset M$ for all $x \in M$. $M \subset X$ is called a minimal set if M is a closed invariant set and does not contain any closed invariant proper subset. Let $M \subset X$ be a closed invariant set (a minimal set). M is said to be isolated from closed invariant sets (minimal sets) if there exists a neighborhood U of M such that U does not contain any closed invariant set (minimal set) except those contained in M. A compact invariant set is said to be positively (negatively) asymptotically stable if for each neighborhood U of M there exists a neighborhood V of M such that $C^+(x)$ $(C^-(x)) \subset U$ for all $x \in V$, and if $\{y \in X; L^+(y) \subset M\}$ $(\{y \in X; L^-(y) \subset M\})$ is a neighborhood of M.

Let X be locally compact, and $M \subset X$ a non-open compact invariant set isolated from closed invariant sets. The following theorem is due to T. Ura and I. Kimura ([17]).

Theorem. *One and only one of the following alternatives holds.*

(1) *M is positively asymptotically stable.*

(2) *M is negatively asymptotically stable.*

(3) *There exist points $x \notin M$ and $y \notin M$ such that $\phi \neq L^+(x) \subset M$*